

# THE ASTROPHYSICAL JOURNAL

An International Review of Spectroscopy and  
Astronomical Physics

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† Died May 9, 1931.

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† Died May 9, 1931.

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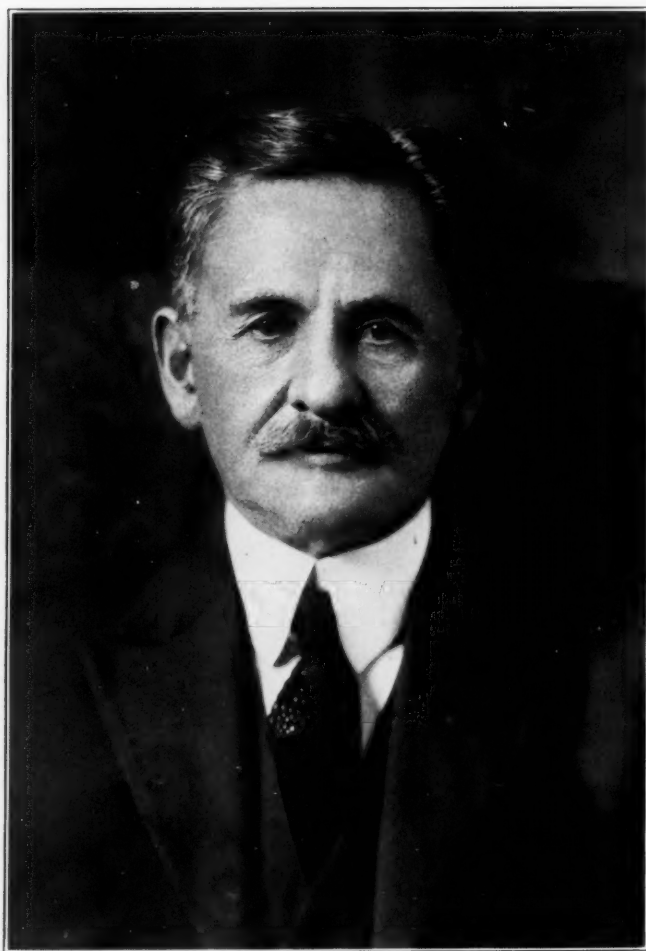
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ALBERT A. MICHELSON  
1852-1931

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## ALBERT A. MICHELSON

By HENRY G. GALE

Professor Albert A. Michelson, head of the Department of Physics at the University of Chicago since 1892, died in Pasadena, California, on May 9, 1931.

He was born in Strelno, Germany, on December 19, 1852. His parents brought him to the United States when he was about two years old and settled in the Far West. Later they located in Virginia City, Nevada, and finally moved to San Francisco where Michelson attended high school. The teacher of science in his high school became very much interested in him and encouraged him to continue his education after the completion of his high-school course. It was decided that he should enter the United States Naval Academy. There was, however, some difficulty about getting an appointment since no vacancies were available. But the young man, having set his heart on entering Annapolis, was not willing to give up without a struggle. He went to Washington and made such a favorable impression upon President Grant and some of the high officials in the navy that he was finally given an appointment to the Naval Academy, in spite of the fact that no vacancies existed. Michelson graduated from the Naval Academy in 1873 and served as midshipman in the navy for two years. On returning from his cruise he was appointed instructor in physics and chemistry in the Naval Academy, a position which he held from 1875 to 1879. The next year was spent in the Nautical Almanac Office in Washington,

and was followed by two years abroad, during which Michelson studied at the College de France and the universities of Heidelberg and Berlin. While he was abroad he received an appointment as professor of physics in the Case School of Applied Science in Cleveland, to begin instruction in the summer of 1882. The trustees of the Case School of Applied Science appropriated \$7500 to be expended under the direction of Professor Michelson for the purchase of physical apparatus. He began his teaching duties in Cleveland in September, 1882. In the summer of 1889 he resigned and became professor of physics at Clark University where he remained three years. He was appointed professor and head of the Department of Physics at the University of Chicago in 1892, a position which he held until he was forced by ill health to resign as professor and became emeritus professor shortly before his death.

Professor Michelson's great contributions to physics were in the field of experimental optics. While he was an instructor at Annapolis, he was ordered by his immediate superior to prepare an experiment to illustrate the measurement of the velocity of light. It is fair to assume that this assignment was the deciding factor of his entire career. With such apparatus as he could get together, he was able to demonstrate the method, and somewhat to his own surprise he found that the results he was getting gave a more accurate value for the velocity of light than any that had yet been obtained. The experiment was described in the *American Journal of Science*,<sup>1</sup> and was Michelson's first scientific publication. In this, and in all later experiments on the velocity of light, he used the revolving-mirror method of Foucault in preference to the rotating toothed wheel of Fizeau. Foucault had used a distance of about 20 m between the rotating mirror and the distant mirror. Michelson by an ingenious rearrangement of the optical parts was able to increase this distance to 700 m, with an almost exactly proportional increase in the accuracy of the measurement. The result was given<sup>2</sup> originally as  $299,940 \pm 50$  km per second. In Michelson's report<sup>3</sup> to Newcomb in 1883 this value is corrected to  $299,910 \pm 50$ .

<sup>1</sup> *Op. cit.* (3d ser.), 15, 394, 1878.

<sup>2</sup> U.S. Nautical Almanac Office, *Astronomical Papers*, 1, Part III, 115, 1880.

<sup>3</sup> *Ibid.*, 2 (1891), Part IV, 231, 1885.

Shortly after coming to Cleveland he undertook a new measurement of velocity of light, aided partly by a governmental appropriation secured through Professor Simon Newcomb and partly through the generosity of Mr. Charles F. Brush, of Cleveland. A level stretch along the tracks of the Nickel Plate Railway at the rear of the Case campus was utilized. Cement piers were erected about half a mile apart to support the mirrors, and the distance between them was measured with great care. Mr. Brush provided a dynamo, operated by a gas engine, and an arc light, accessories which were by no means common at that time. The value obtained in these experiments at Cleveland in 1883 was  $299,853 \pm 60$  km per second, a value accepted as standard for many years. These results were reported in 1883.<sup>1</sup> The "probable error" was  $\pm 12$  km per second, the estimated maximum error,  $\pm 60$ . Newcomb's result,  $299,860 \pm 30$ , completed in 1882, was in substantial agreement with Michelson's.

The original purpose of Foucault's experiment was to test the question of the relative velocities of light in air and water.

The experiment by Foucault appears to have given merely a satisfactory qualitative result, but Professor Michelson, in 1883 and 1884, showed not only that the velocity of light is less in water than in air, but that the ratio of the velocities was almost exactly equal to the index of refraction, as was to be expected on the wave theory of light. But when water was replaced by carbon disulphide, the ratio of the velocities was found to be 1.75, instead of 1.64, the index of refraction. It was very characteristic of Professor Michelson that he did not hesitate to publish this apparently contradictory result, but did so with fine confidence in the reliability of his own measurements. Lord Rayleigh showed at once that the result is the one to be expected since the revolving-mirror experiment, like that with the toothed wheel, gives the velocity of isolated groups of wave trains, which is not the same as the wave velocity except in a medium without dispersion, and that the ratio of the velocity in air to the velocity in carbon disulphide should be  $1.64/0.93 = 1.76$ , in agreement with the result obtained. Michelson also showed at this time, using a tube of carbon disulphide about 10 feet long, that red light

<sup>1</sup> *Loc cit.*, 2 (1891), Part IV, 231, 1885.

traveled about 2 per cent faster than blue light. This result is important in theories of dispersion.

Between 1885 and 1889 Professor Michelson collaborated with Edward W. Morley, professor of chemistry in Western Reserve University, in three important investigations. In 1886 they showed that a moving water column affects the velocity of light, not by the full amount of the velocity of the water, but by less than half of this velocity  $[(n^2 - 1)/n^2]$  as predicted by the theory of Fresnel.

In 1887 Michelson and Morley performed the famous ether-drift experiment. This was one of the best-known experiments in which was used the Michelson interferometer,<sup>1</sup> an instrument designed to show interference between two beams of light traveling at right angles to each other. The ether-drift experiment was expected to show a displacement of about 0.4 fringe when the interferometer was rotated through 90°, but the displacements were certainly not more than 0.01 of a fringe. As is well known, the negative character of their results was largely responsible for the development of the theory of relativity, by Lorentz, Fitzgerald, Einstein, and others.

In 1888 and 1889 Michelson and Morley published their method for determining the number of wave-lengths of light in the standard meter. This was another application of the interferometer, and amounted to making light-waves, instead of an arbitrary bar, the standard of length.

In the search for a suitable light-wave, in terms of which to evaluate the meter, Professor Michelson discovered the fine structure of spectral lines. The double character of the red hydrogen line was a discovery which has had a profound influence on the development of modern theories of the nature of the atom. The determination of the number of the wave-lengths of the red cadmium line in a meter, or the wave-length of the red cadmium line, 6438.4722 Å, was a masterpiece of skill, and the result still stands for air of normal density and of average humidity.

Michelson was an observer of extraordinary skill, and a physicist of remarkable intuitive power. These characteristics are nowhere better illustrated than in his studies of visibility-curves. His naked-eye observations, estimating the relative intensities of the bright

<sup>1</sup> *Philosophical Magazine* (5th ser.), 13, 236, 1882.

and dark fringes, and plotting the visibility-curves for increasing differences of path, were as exact as a skilled observer could make with the best mechanical aids. His theory and interpretations of the visibility-curves added an interesting and important chapter to the development of modern spectroscopy.

During his three years at Clark University, 1889-1892, he was chiefly active in pushing his investigations with the interferometer. It was in 1891 that he published his method for measuring stellar diameters, and actually applied it to the measurement of some of Jupiter's satellites at the Lick Observatory. He never lost interest in this problem, and often spoke of a desire to take it up on a larger scale, but it was not until 1920 at the Yerkes Observatory, and 1921 at Mount Wilson, that he saw the fulfilment of his wishes, climaxed by the measurement, with F. G. Pease, of the diameter of  $\alpha$  Orionis in 1921.

Professor Michelson became head of the Department of Physics at the University of Chicago in 1892, but was granted a year's leave of absence to complete his evaluation of the meter in terms of the wave-length of the cadmium lines, at the Bureau International des Poids et Mesures. His *Valeur du mètre* appeared in 1894. It contains the theory of the interferometer and a résumé of his work on visibility-curves in addition to a complete account of his comparison of the international meter with the wave-length of the light of cadmium. During the next few years he published a number of papers of astrophysical interest, a few on X-rays, then in their infancy, and an account of an ether-drift experiment with a large vertical interferometer. This yielded a negative result, as did the Michelson-Morley experiment.

Professor Michelson was intensely interested by Zeeman's discovery that spectral lines are influenced by a magnetic field. He showed, by the use of the interferometer, that the lines were not merely broadened, and polarized at the edges, but were actually broken up into separated components. It was largely through a desire to secure sufficiently high resolving power to show this effect directly, none too easy to observe with the comparatively small electromagnets then available, that he invented the echelon spectro-scope, which served a splendid purpose later in the study of fine

structure and the Zeeman effect. It was in connection with the analysis of the visibility-curves arising from the Zeeman lines that he and Professor S. W. Stratton designed a new harmonic analyzer, with eighty elements. A few years later he began to work on a new ruling engine, for the making of diffraction gratings. He succeeded in making one 10-inch and one 8-inch plane grating of excellent quality, and a considerable number of smaller gratings.

Professor T. C. Chamberlin had often urged Professor Michelson to investigate the earth tides, as an indication of the rigidity of the earth. The best methods then in use had given rather uncertain results. With characteristic skill Professor Michelson designed a new method, and one capable of giving results of considerable accuracy. An iron pipe, 6 inches in diameter and 500 feet long, was placed underground, and half filled with water. It was possible to measure the tides in this pipe quite accurately, and from them to deduce the tides in the solid earth.

An experiment which Professor Michelson had wished for years to try was performed at Clearing, Illinois, in 1925. For many years he had mentioned in lectures to his classes the possibility of testing the velocity of light in an east-west path as compared with a west-east path, by the device used in the Clearing experiment. A beam of light was divided, and the parts sent around a rectangular path in opposite directions, to recombine and interfere at the dividing plate. The same experiment had been suggested by Sir Oliver Lodge. The final result, a displacement of about a quarter of a fringe, might be interpreted as an argument in favor of relativity, or equally well, as an argument for a fixed ether, but excluded the possibility of ether drift.

It was during these years, from 1924 on, that Professor Michelson again took up the experimental determination of the velocity of light. At the invitation of Dr. George E. Hale, then director of the Mount Wilson Observatory, Professor Michelson became a research associate of the Carnegie Institution of Washington, and began to spend his vacations in California. It was in this capacity that he determined the diameter of Betelgeuse with Mr. Pease, in 1925, and measured the velocity of light between Mount Wilson and Mount San Antonio, 22 miles away, in several successive years. The results

for 1924<sup>1</sup> gave, for the velocity *in vacuo*,  $V = 299,820$ . This was corrected in 1927 to 299,802. The results for 1925 were  $V = 299,756$ , and a third series gave  $V = 299,771$ . The weighted mean for the velocity *in vacuo* was  $V = 299,771$ .

Observations with the same equipment in the summer of 1926 gave the following remarkable series of values:<sup>2</sup>

Turns per Second	Mirror	Number Obs.	Vel. of Light <i>in Vacuo</i>
528.....	Glass octagon	576	299,797
528.....	Steel octagon	195	299,795
352.....	Glass 12	270	299,796
352.....	Steel 12	218	299,796
264.....	Glass 16	504	299,796

Not satisfied with this superb result, Professor Michelson had worked since 1926 to perfect a method for measuring the velocity of light in a 3-foot pipe, a mile long, which could be exhausted. This experiment has been successful and the results will soon be ready for publication. He frequently referred to this as his last experiment, and so it has become. But those who knew him best feel sure that it would have been followed by many others if he had lived, even though suffering from desperately bad health.

He was an optimist. In a letter dictated two days before he became unconscious he said, "My health continues to improve," and he outlined plans for the continuation and extension of the work on the velocity of light.

The most characteristic thing about his work was that he selected big ideas. He liked to play, and he did play in the laboratory. He has perhaps done more simple experiments and then forgotten all about them and never published them than most men have done. He lost interest in them because he did not think they were big enough. The things which interested him were the big things. He scorned to publish anything that he regarded as beneath the high standard he set himself. He published about seventy-five papers, all of them classics.

His point of view toward science is best illustrated by quoting some of the things which he himself wrote; for example:

<sup>1</sup> *Astrophysical Journal*, 60, 256, 1924.

<sup>2</sup> *Ibid.*, 65, 1, 1927.

It seems to me that scientific research should be regarded as a painter regards his art, a poet his poems, a composer his music. It would be quite as unfair to ask of these an apology for their efforts; and the kind of benefit which I should most appreciate from research in pure science is much more allied to such non-material results, results which help to increase the pleasure of our all too small matter of fact existence, and which help to teach man his true relation to his surroundings—his place in nature.

In a somewhat lighter vein he wrote about the ruling engine on which he worked for many years:

One comes to regard the machine as having a personality—I had almost said a feminine personality—requiring humoring, coaxing, cajoling, even threatening! But finally one realises that the personality is that of an alert and skilful player in an intricate but fascinating game who will take immediate advantage of the mistakes of his opponent, who “springs” the most disconcerting surprises, who never leaves any result to chance, but who nevertheless plays fair, in strict accordance with the rules of the game. These rules he knows, and makes no allowance if you do not. When *you* learn them, and play accordingly, the game progresses as it should.

And finally:

Doubtless to the lay mind and certainly to the practical man of affairs, upon whom unfortunately we are dependent for support, the argument of the practical value of scientific research will appeal more powerfully than the true reason for the activities of the investigator—namely, the love of the work for its own sake.

When asked why he wanted to do this last experiment on the velocity of light, he made some half-hearted explanation about the value to science, and then he added laughingly, “But the real reason is because it is such good fun.”

He was extremely fond of tennis and he was a good player. He played two seasons in the international tennis matches at Newport and regularly at the Kenwood Country Club after he came to Chicago, and at the Quadrangle Club until after he was seventy years of age. He was a good violinist, and a fine billiard player. He enjoyed bridge and chess and played often at the University Club and at the Quadrangle Club. He found perhaps his greatest pleasure, however, in sketching, and he always took his water-color kit with him when he went on a vacation trip.

He received most of the honors which can come to a scientific man, membership in scientific societies, honorary degrees, and medals for scientific merit. The following list is nearly if not quite complete:

Corresponding Member, British Association for the Advancement of Science, 1884; Ph.D. (hon.), Western Reserve University, 1886, and Stevens Institute, 1887; Vice-President, American Association for the Advancement of Science, 1887; Member, National Academy of Science, 1888; Rumford Medal, 1889; Bureau International des Poids et Mesures, 1892-1893. Member, Société Française de Physique, 1893; Associate, Royal Astronomical Society, 1894; Foreign Member, Société Hollandaise des Sciences, 1897; Honorary Member, Cambridge Philosophical Society, 1897; Member (for the United States), International Committee of Weights and Measures, 1897; Sc.D. (hon.), Cambridge, 1899; Honorary Member, Royal Institution, 1899; Membre correspondant de l'Académie des Sciences, Paris, 1900; Grand Prix, Exposition Générale de Paris, 1900; President, American Physical Society, 1900; LL.D., Yale University, 1901; Member, American Philosophical Society, 1902; Fellow, Royal Society, 1902; Matteucci Medal, Soc. Italiana, Rome, 1904; LL.D., University of Pennsylvania, 1906; Member, Kungliga Vetenskaps Akademien, Stockholm, 1906; Member, Reale Accademia dei Lincei, Rome, 1906; Copley Medal, 1907; Nobel Prize, 1907; Hon. Member, Royal Irish Academy, 1908; Ph.D., University of Leipzig, 1909; President, American Association for the Advancement of Science, 1910-1911; Ph.D., Georg-August University, Göttingen, 1911; Member, Deutsche Physikalische Gesellschaft, Berlin, 1911; Member, Kungliga Fysiografiska Sällskapet, Lund, 1911; Ph.D., Royal Frederick University, Christiania, 1911; Elliott Cresson Medal, Franklin Institute, 1912; Draper Medal, National Academy of Science, 1916; Foreign Associate, Académie Française, 1920; Ph.D., University of Paris, 1921; Honorary Fellow, Optical Society, 1921; Honorary Member, Société de Physique, 1921; LL.D., McGill University, 1921; Franklin Medal, 1923; Gold Medal, Royal Astronomical Society, 1923; Honorary Member, Franklin Institute, 1923; President, National Academy of Science, 1923; Associate, Académie Royale de Belgique, 1923; Distinguished Service Professor of Physics, University of Chicago, 1925; Member, Russian Academy of Sciences, 1926; Sc.D., Princeton University, 1927; Gold Medal of the Society of Arts and Sciences (N.Y.), February 22, 1929; Duddell Medal of the Physical Society of London, 1930.

It is not always that a man of Professor Michelson's undoubted genius is granted so liberal a span of life. He would have been seventy-nine years old if he had lived until next December, and his friends and the scientific world must experience a profound feeling of gratitude that he maintained his fine mental powers and his alert interest in research until two days before his death. New theories in physics will come and go, but his splendid contributions of exact quantitative measurements have become an integral part of physics, and will be an example and an inspiration to future generations of physicists.

# A SPECTROPHOTOMETRIC STUDY OF $\eta$ AQUILAE

By C. J. KRIEGER

## ABSTRACT

The investigation is based on three spectrograms of  $\eta$  Aquilae by C. S. Yü in 1926 and seventeen spectrograms obtained by the author in 1929 at the Lick Observatory. Moll microphotometer tracings of all plates were made.

*Spectral type.*—The spectral type is found to vary between F2 and G5. The corresponding temperature change, considering  $\eta$  Aquilae as a normal giant, is from 4700° to 7000° K.

*Variation of surface temperature.*—Mendenhall's method, by the use of  $\gamma$  and  $\delta$  Aquilae as comparison stars, furnishes a variation from 4200° to 7100°. The variation based on color indices is from 4050° to 6850°. The mean variation of the effective surface temperature is from 4100° to 7000°, with a p.e. of an individual determination of  $\pm 200^\circ$ .

*Variation of magnitude.*—The magnitude curves at  $\lambda\lambda$  4200, 4500, and 4800 are derived. The p.e. of an individual determination is  $\pm 0.12$ .

*Line intensities.*—The strengths of the  $\beta$ ,  $\gamma$ , and  $\delta$  lines of hydrogen, the H and K lines, and the G band are determined in terms of the intensity of the continuous background.

## INTRODUCTION

The Cepheid variables have for a long time attracted considerable attention, partly because of their light variation, radial velocity, and spectral changes which challenged the observers to account for these three simultaneous periodic processes as different aspects of a single periodic phenomenon; partly also because they are the basis of a method of measuring distance which has proved exceedingly fruitful. With the development of spectrophotometric methods certain Cepheid variables were studied in regard to the distribution of the intensity of the continuous background, line and band intensities, and changes in spectral types.<sup>1</sup>

It has seemed desirable to utilize some of the newer methods for the determination of the variation of the surface temperature of  $\eta$  Aquilae, and to make a quantitative study of the variations of line intensities from the spectrophotometric data.

## I. OBSERVATIONS

During the summer of 1929 a series of observations of  $\eta$  Aquilae was made with the two-prism slitless quartz spectrograph attached

<sup>1</sup> Ch'ing-Sung Yü, *Publications of the Astronomical Society of the Pacific*, **38**, 357, 1926; H. S. Mendenhall, *Lick Observatory Bulletin*, **14**, 133, 1930; Ch'ing-Sung Yü, *ibid.*, **15**, 1, 1930.

to the Crossley Reflector at the Lick Observatory (Table I). Eastman 40 plates, developed in D-61, were used throughout.

The observations of  $\eta$  Aquilae were taken with a diaphragm having four circular openings of ten-inch diameter. The exposure time was one minute.

On each plate three spectrograms of one comparison star were obtained, with the same exposure time as that of the variable (see

TABLE I  
OBSERVATIONS OF  $\eta$  AQUILAE

Plate	Phase	Date 1929	Sid. T.	J.D. 2425000+	Comp. Star.
238	0 <sup>d</sup> .27	July 14	17 <sup>h</sup> 16 <sup>m</sup>	807.744	$\delta$ Aql
217	0.48	July 7	17 <sup>h</sup> 24 <sup>m</sup>	800.770	$\delta$ Aql
218	0.48	July 7	17 <sup>h</sup> 30 <sup>m</sup>	800.774	$\gamma$ Aql
222	1.47	July 8	17 <sup>h</sup> 10 <sup>m</sup>	801.758	$\delta$ Aql
223	1.47	July 8	17 <sup>h</sup> 20 <sup>m</sup>	801.767	$\gamma$ Aql
243	1.92	July 30	18 <sup>h</sup> 12 <sup>m</sup>	823.740	$\delta$ Aql
244	1.92	July 30	18 <sup>h</sup> 14 <sup>m</sup>	823.741	$\gamma$ Aql
226	2.44	July 9	16 <sup>h</sup> 42 <sup>m</sup>	802.734	$\delta$ Aql
229	3.58	July 11	20 <sup>h</sup> 00 <sup>m</sup>	803.875	$\gamma$ Aql
230	3.59	July 11	20 <sup>h</sup> 02 <sup>m</sup>	803.876	$\delta$ Aql
249	3.93	Aug. 1	18 <sup>h</sup> 34 <sup>m</sup>	825.750	$\delta$ Aql
250	3.93	Aug. 1	18 <sup>h</sup> 36 <sup>m</sup>	825.751	$\gamma$ Aql
233	4.45	July 11	16 <sup>h</sup> 58 <sup>m</sup>	804.740	$\gamma$ Aql
234	4.45	July 11	17 <sup>h</sup> 00 <sup>m</sup>	804.741	$\delta$ Aql
212	5.71	July 5	18 <sup>h</sup> 28 <sup>m</sup>	798.819	$\gamma$ Aql
213	6.69	July 6	18 <sup>h</sup> 09 <sup>m</sup>	799.804	$\delta$ Aql
214	6.70	July 6	18 <sup>h</sup> 14 <sup>m</sup>	799.806	$\gamma$ Aql

Fig. 1). The first exposure was taken with four ten-inch circles open, the second with two ten-inch circles open, and the third with one ten-inch circle open, immediately preceding or following the exposure of the variable which was taken with the four circles open. The positions and characteristics of both the variable star  $\eta$  Aquilae and the comparison stars employed are:

$\eta$  AQUILAE, Boss No. 5071, H.D. No. 187929,  $\alpha$  (1900)=19<sup>h</sup>47<sup>m</sup>.4,  $\delta$  (1900)=+0°45',  $P=7^d$ 177, Max. Phase, J.D. 2414827.15 G.M.T.<sup>1</sup>

$\gamma$  AQUILAE, Boss No. 5047, H.D. No. 186791,  $\alpha$  (1900)=19<sup>h</sup>41<sup>m</sup>.5,  $\delta$  (1900)=+10°22', gK2, Pm. Mag. 2.80, Pg. Mag. 3.87,  $T=3900^\circ$

$\delta$  AQUILAE, Boss No. 4953, H.D. No. 182640,  $\alpha$  (1900)=19<sup>h</sup>20<sup>m</sup>.5,  $\delta$  (1900)=+2°55', Fo, Pm. Mag. 3.44, Pg. Mag. 3.72,  $T=7400^\circ$

<sup>1</sup> *Kl. Veröffentlichungen der Universitätssternwarte zu Berlin-Babelsberg*, No. 5, 1929.

## II. ESTIMATES OF SPECTRAL TYPES AND TEMPERATURES

The spectral types were estimated on the basis of the Henry Draper system of classification, and temperatures were assigned

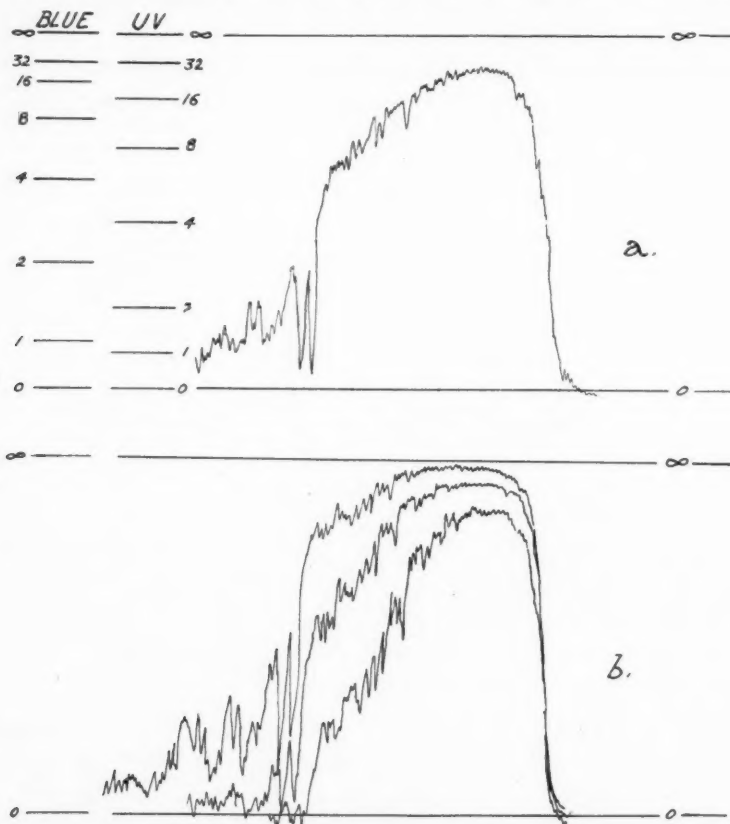


FIG. 1.—Microphotometric tracing of spectrum of  $\eta$  Aquilae (a) and of comparison star  $\gamma$  Aquilae (b).

from the relation established between spectral types and temperatures.<sup>1</sup> For this purpose the Cepheid variables were considered as normal giants, since the table does not give the relation for supergiants. This is to be borne in mind when comparing the temperatures thus obtained with those derived from the distribution of intensity of the continuous background or from color-indices. However, there

<sup>1</sup> Russell, etc., *Astronomy*, 2, 753, 1927.

is good agreement with those derived by Harlow Shapley,<sup>1</sup> also from estimates of spectral types.

The three plates Y 614, Y 617, and Y 620 had been taken by C. S. Yü on October 6, 7, 18, 1926.

TABLE II  
ESTIMATES OF SPECTRAL TYPES OF  $\eta$  AQUILAE  
(See Fig. 2)

Plate	Phase	Sp. Type	Temp. °K
Y 614.....	0 <sup>d</sup> 05	F2	7000°
238.....	0.27	F2	7000
217.....	0.48	F3	6830
218.....	0.48	F3.5	6720
Y 617.....	1.07	F4	6620
222.....	1.47	F5	6500
223.....	1.47	F6	6270
243.....	1.92	F5	6500
244.....	1.92	F5	6500
226.....	2.44	F6	6270
229.....	3.58	F7	6100
230.....	3.59	F8	5920
249.....	3.92	F9	5740
250.....	3.92	F9	5740
233.....	4.44	G3	5100
234.....	4.45	G2	5230
Y 620.....	4.89	G5	4700
212.....	5.71	F9	5740
213.....	6.69	F3	6830
214.....	6.70	F4	6620

### III. INTENSITY DISTRIBUTION OF THE CONTINUOUS BACKGROUND AND BLACK-BODY TEMPERATURES

The Moll self-registering microphotometer furnished tracings about 120 mm long, the distance of the lamp and that of the thermocouple from the plate being 200 and 120 mm, respectively. The voltage of the lamp was kept constant at 4.5 volts. Only the central part of the spectrum was measured by attaching a narrow horizontal slit in front of the vertical thermocouple slit, thus avoiding the admission of light falling through the clear plate at the edges of the spectrum. Two sets of sensitometric exposures were impressed on the photographic plate before development, the intensities being in the ratio 1, 2, 4, 8, 16, 32. The first series was made through an ultra-violet filter, Wratten No. 18, with a maximum transmission at

<sup>1</sup> *Astrophysical Journal*, 44, 283 and 287, 1916.

$\lambda$  3600, and the second series through a blue filter, Wratten No. 50, with a maximum transmission at  $\lambda$  4600.

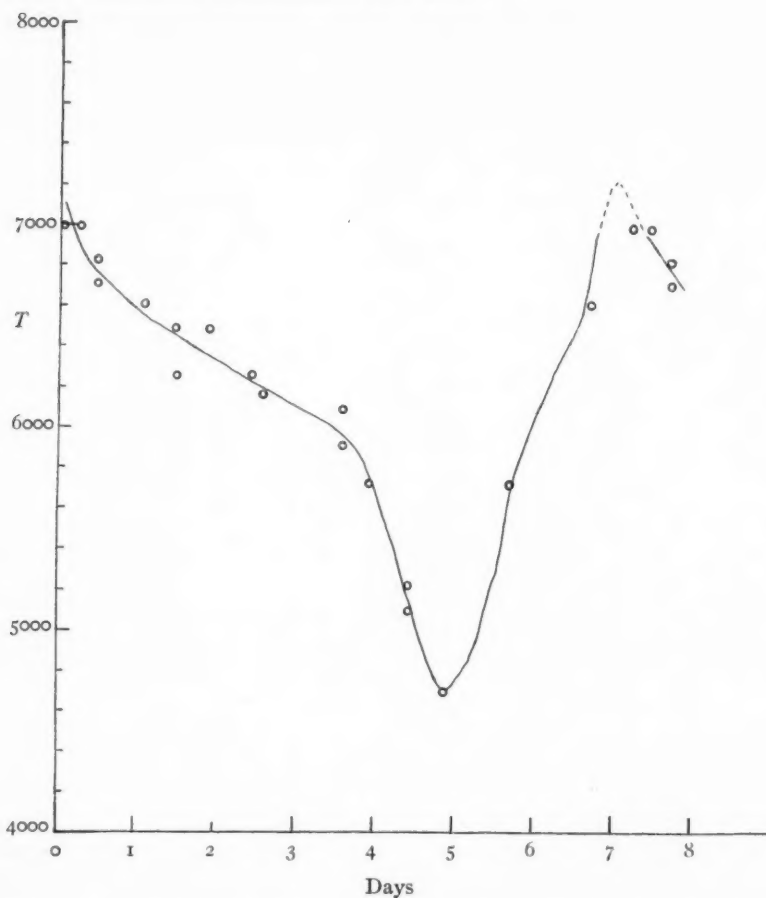


FIG. 2.—Temperature variation of  $\eta$  Aquilae based on estimates of spectral type

a) METHOD USED BY H. S. MENDENHALL

In a recent publication<sup>1</sup> Mendenhall has described in detail a method for the spectrophotometric observation of variation of magnitude. The spectrum of the variable is obtained with several spectra of a comparison star on the same plate, and with the same exposure time. The aperture of the telescope is adjusted so that the

<sup>1</sup> *Loc. cit.*

intensity of the spectrum of the variable is intermediate between the faintest and brightest comparison spectrum. At any desired wavelength the intensity-blackening curve is then available, and the

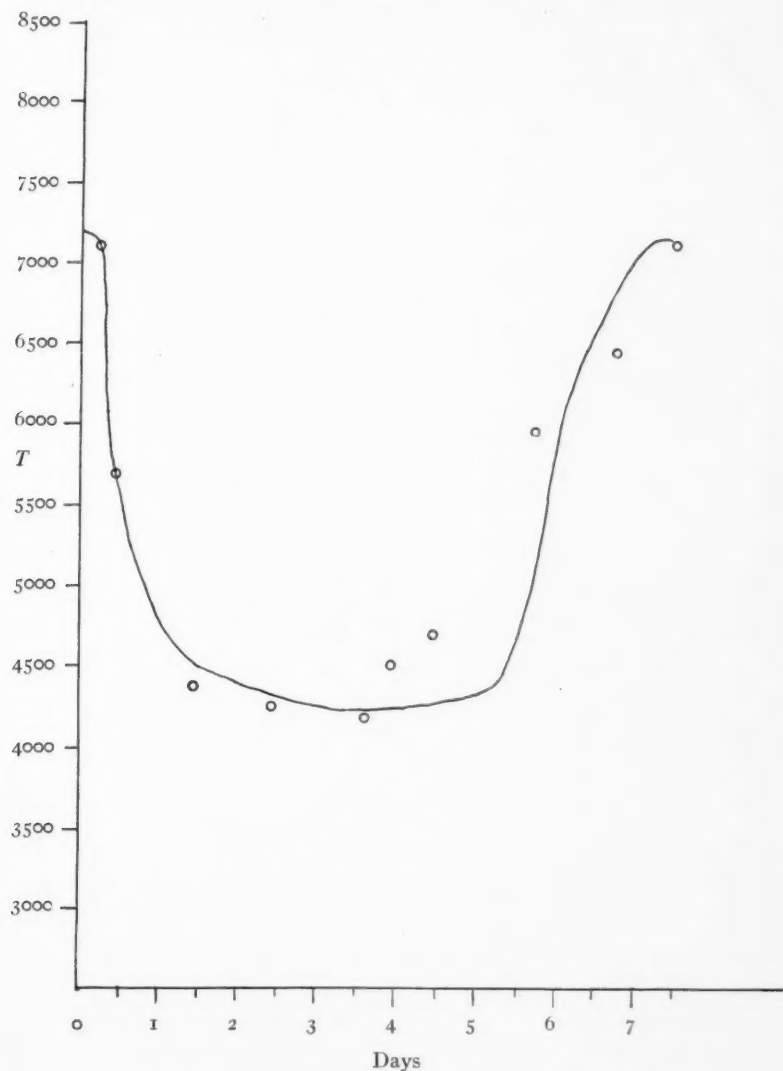


FIG. 3.—Temperature variation of  $\eta$  Aquilae from fitting of Planck's curves. Comparison stars,  $\gamma$  Aquilae and  $\delta$  Aquilae.

TABLE III  
BLACK-BODY TEMPERATURE AND MAGNITUDE VARIATION OF  $\eta$  AQUILAE  
BASED ON COMPARISON STARS  $\gamma$  AQUILAE AND  $\delta$  AQUILAE

PHASE	PLATE	TEMP. BASED ON COMPARISON STARS		MEAN TEMP. °K	MAGNITUDE VARIATION		
		$\gamma$ Aql	$\delta$ Aql		$\lambda$ 4200	$\lambda$ 4500	$\lambda$ 4800
0 <sup>d</sup> 27.....	238	.....	7100°	7100°	4.43	4.40	4.36
0.48.....	217	.....	5700	5700	4.16	4.08	4.01
1.47.....	222	.....	4300	4375	4.66	4.36	4.19
1.47.....	223	4450	.....		.....	3.98	3.85
2.44.....	226	.....	4250	4250	4.54	4.23	4.28
3.58.....	229	4730	.....	4190	4.42	4.28	4.12
3.59.....	230	.....	3650		5.26	4.90	4.68
3.93.....	249	.....	4500	4500	5.08	4.82	4.80
4.45.....	233	5200	.....	4700	4.70	4.66	4.50
4.45.....	234	.....	4200		5.27	4.95	4.82
5.71.....	212	5960	.....	5960	4.67	4.67	4.63
6.70.....	214	6450	.....	6450	.....	4.10	4.03

TABLE IV  
VARIATION OF MAGNITUDE AND TEMPERATURES FROM  
COLOR-INDEX OF  $\eta$  AQUILAE

PHASE	MAG.			C.I.			C.I. TEMP.			AVE. C.I. TEMP.	BLACK- BODY TEMP.	MEAN TEMP. (2 DET.)
	$\lambda$ 4200	$\lambda$ 4500	$\lambda$ 4800	$\lambda\lambda$ 4200- 4500	$\lambda\lambda$ 4500- 4800	$\lambda\lambda$ 4200- 4800	$\lambda\lambda$ 4200- 4500	$\lambda\lambda$ 4500- 4800	$\lambda\lambda$ 4200- 4800			
0 <sup>d</sup> 0...	4.03	4.07	4.03	-.04	.04	0	7500°	5550°	6600°	6550°	7200°	6900°
0.5...	4.25	4.12	4.04	.13	.08	.21	4900	5000	4950	4950	5600	5300
1.0...	4.35	4.16	4.05	.19	.11	.30	4400	4650	4500	4500	4750	4600
1.5...	4.46	4.20	4.10	.26	.10	.36	3950	4750	4250	4300	4500	4400
2.0...	4.55	4.25	4.17	.30	.08	.38	3700	5000	4200	4300	4400	4300
2.5...	4.70	4.35	4.28	.35	.07	.42	3450	5100	4050	4200	4300	4300
3.0...	4.83	4.52	4.42	.31	.10	.41	3950	4750	4100	4250	4250	4200
3.5...	4.94	4.64	4.52	.30	.12	.42	3700	4550	4050	4100	4220	4200
4.0...	5.03	4.75	4.62	.28	.13	.41	3800	4450	4100	4100	4230	4200
4.5...	5.13	4.85	4.70	.28	.15	.43	3800	4300	4000	4050	4250	4100
5.0...	5.07	4.93	4.73	.14	.20	.34	4800	3900	4350	4350	4300	4300
5.5...	4.78	4.77	4.66	.01	.11	.12	6400	4650	5500	5500	4700	5100
6.0...	4.50	4.52	4.49	-.02	.03	.01	7000	5700	6500	6400	5900	6100
6.5...	4.23	4.27	4.22	-.04	.05	.01	7500	5400	6500	6450	6650	6500
7.0...	4.03	4.10	4.05	-.07	.05	-.02	8300	5400	6800	6850	7050	7000

difference of magnitude of the variable and comparison star can be determined, after correction for differential atmospheric absorption. By assigning a value of the temperature to the comparison star from the known spectral class and absolute magnitude, the rela-

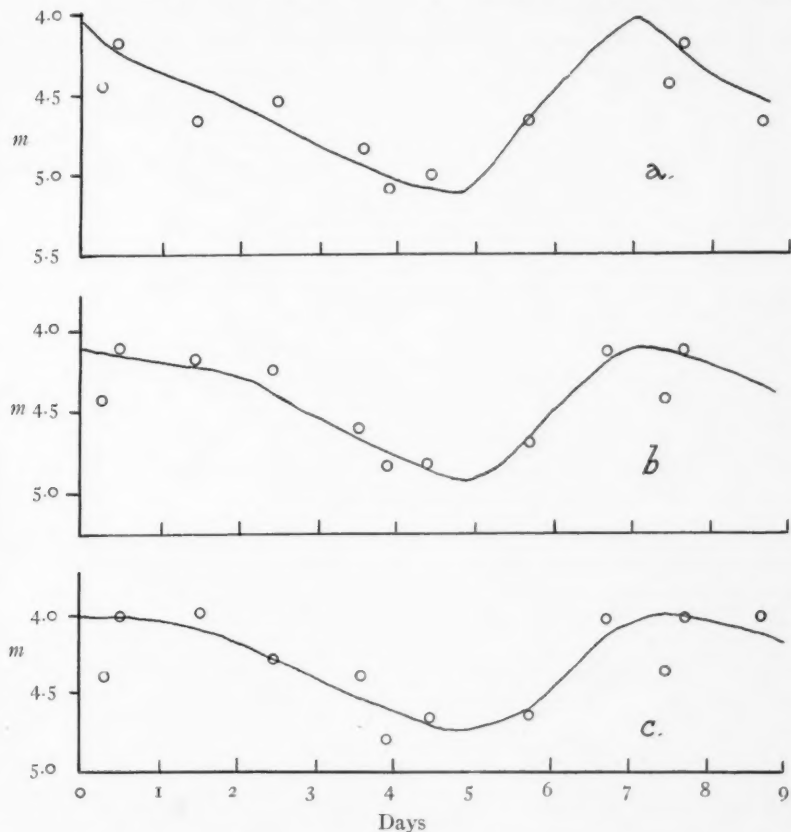


FIG. 4.—Variation in magnitude of  $\eta$  Aquilae at: (a),  $\lambda 4200$ ; (b),  $\lambda 4500$ ; (c),  $\lambda 4800$

tion of magnitude and wave-length of the variable is derived by combining the magnitude difference between the variable and the comparison star, with the black-body curve of the comparison star, expressed in the magnitude scale.

As an illustration, Plate 229, at phase  $3^d 58$ , is discussed briefly.

To the three spectra of the comparison star with 1, 2, and 4 holes opened are assigned 1.50, 0.75, and 0.00 mag., respectively. For each

wave-length investigated the blackening-magnitude curve may then be drawn from three points, and the magnitude difference between variable and comparison star,  $\Delta m$ , may be found from the known blackening of the variable.

The correction for differential atmospheric absorption caused by the difference in zenith distance of variable and comparison star is applied as  $2.5 (\sec z_1 - \sec z_2) \log a_{0\lambda}$ . The corrected  $\Delta m$  expresses

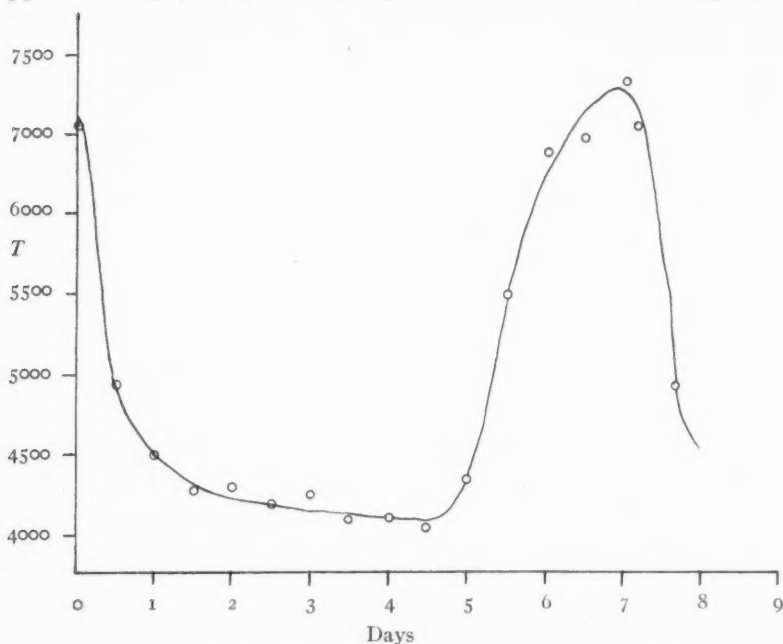


FIG. 5.—Temperature variation of  $\eta$  Aquilae from color-indices

the magnitude difference between  $\eta$  Aquilae and  $\gamma$  Aquilae. As the black-body temperature of  $\gamma$  Aquilae was assumed as  $3900^\circ$ , and as its photographic magnitude (at  $\lambda 4250$ ) is 3.87, the relationship of magnitude and wave-length of  $\gamma$  Aquilae is given, and upon adding  $\Delta m$ , also that of  $\eta$  Aquilae. The fitting of Planckian curves through the points  $\lambda\lambda 4000, 4200, 4400, 4600$ , and  $4800$  indicates a temperature of about  $4700^\circ$ .

The temperatures derived in this manner from comparison stars  $\gamma$  Aquilae and  $\delta$  Aquilae are shown in columns 3 and 4, Table III, and the mean in column 5 (see Fig. 3).

## b) VARIATION OF MAGNITUDE AND COLOR-INDEX TEMPERATURES

The variations of the magnitude at wave-lengths  $\lambda\lambda$  4200, 4500, and 4800 are shown in columns 6, 7, and 8, Table III, and are represented graphically in Figure 4*a*, *b*, and *c*. From the smoothed curves, columns 2, 3, and 4, Table IV, the magnitude differences at  $\lambda$  4200

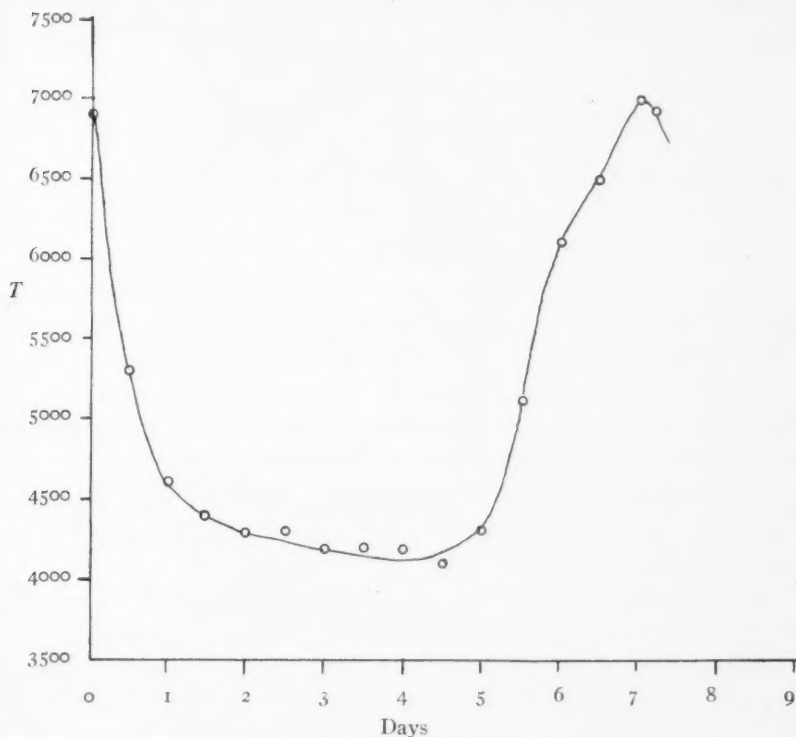


FIG. 6.—Mean temperature variation of  $\eta$  Aquilae. Average of determinations from fitting Planck's curves and from color-indices.

and  $\lambda$  4500,  $\lambda$  4500 and  $\lambda$  4800, and  $\lambda$  4200 and  $\lambda$  4800 were obtained (cols. 5, 6, and 7) and the corresponding temperature derived (cols. 8, 9, 10, and 11). The average is given in Figure 5.

The mean of the black-body temperature (derived in Table III) and of the color-index temperature is shown in the last column of Table IV (see Fig. 6).

A comparison of Figure 6 with the temperatures based on estimates of spectral type (Fig. 2) indicates for the former a more rapid

drop in the descending branch, and, in general, lower temperatures. Apparently the same degree of ionization, to judge from the spectral types, occurs for the variable at lower temperatures than for normal giants. This would require, however, a lower pressure in the stellar atmosphere. Saha's equation furnishes the logarithm of the ratio of the electronic pressures required to produce the same amount of ionization for two different temperatures  $T_1$  and  $T_2$ :

$$\log \frac{P_1}{P_2} = 5.048 \cdot I \left( \frac{1}{T_2} - \frac{1}{T_1} \right) + 2.5 \log \frac{T_1}{T_2},$$

where  $T_2$  is the temperature from the distribution of intensity of the continuous background (Table IV) and  $T_1$  is the temperature from estimates of the spectral type (Table II).

For an ionization potential of 6 volts,<sup>1</sup> a pressure variation involving a ratio of the order  $10^2$  and  $10^3$  would result during a cycle, with minimum pressure  $P_2$  occurring around phase 2<sup>d</sup>.0 or 3<sup>d</sup>.0, and maximum pressure  $P_2$  near phase 6<sup>d</sup>.0. It is interesting to note that the maximum radius computed from the observed magnitudes and temperatures by a method described by W. Baade<sup>2</sup> occurs near phase 2<sup>d</sup>.0, and minimum radius near 7<sup>d</sup>.0. If the atmosphere were pulsating, maximum radius and minimum pressure would be expected to coincide, or nearly so.

Tiercey<sup>3</sup> found a similar variation of pressure by a different method, with a pressure ratio of about 4:1 during a cycle. Maximum pressure and minimum radius, also minimum pressure and maximum radius coincide very nearly in his case.

The probable error of an individual determination of temperature and magnitude was found to be  $\pm 200^\circ$  and  $\pm 0^m.12$ , respectively.

#### IV. LINE INTENSITIES

In the manner described by Yü<sup>4</sup> the strengths of the  $\beta$ ,  $\gamma$ , and  $\delta$  lines of hydrogen, the H and K lines, and the G band were deter-

<sup>1</sup> B. Sticker, *Zeitschrift für Physik*, **61**, 562, 1930: Temperaturen von Riesen und Zwergsternen.

<sup>2</sup> *Astronomische Nachrichten*, **228**, 359, 1926; see also G. Tiercey, *Archives des sciences physiques et naturelles*, **12**, 193, 1930.

<sup>3</sup> *Op. cit.*, p. 200, 1930.

<sup>4</sup> *Lick Observatory Bulletin*, **15**, 6, 1930.

mined in terms of the intensity of the continuous background. The percent-blackening of the center of the line and of the background was measured and expressed in terms of intensity. Since the factor

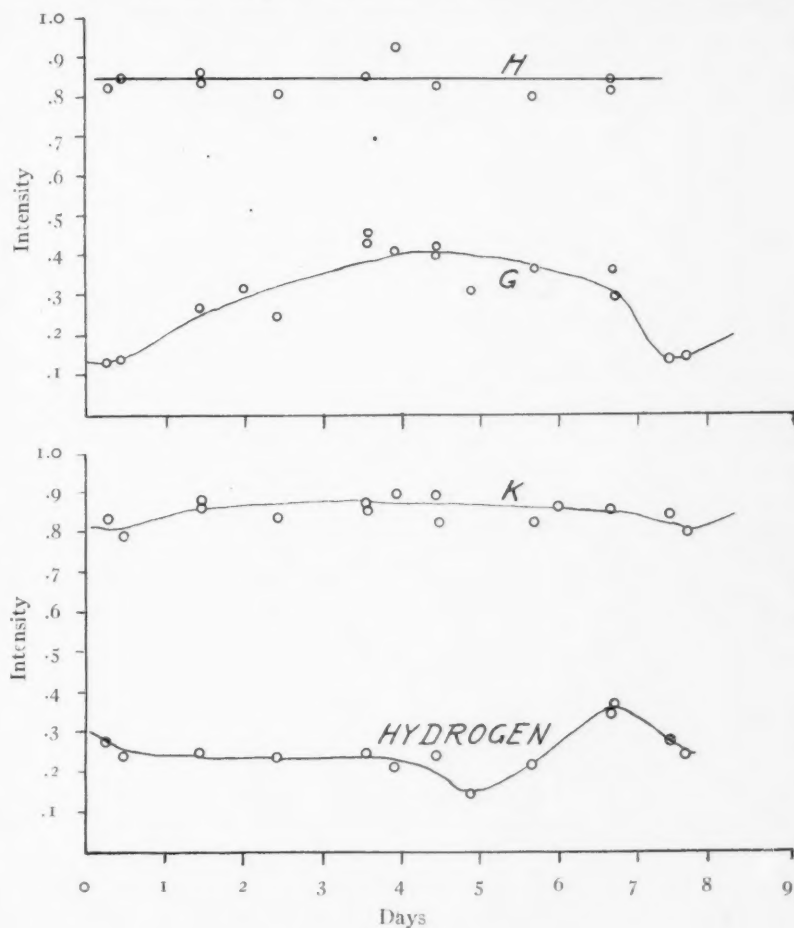


FIG. 7.—Variation of line-intensities

of atmospheric absorption  $a_\lambda$ , and the correction factor  $(rbds)_\lambda$  is the same for both the center of the line and background at any one wave-length, the ratio of the apparent intensities is identical with the ratio of the true intensities. The difference in intensity between the background and the line furnishes the strength of the absorption

line, and by dividing it by the intensity of the background, the percentage loss of light is obtained.

Table V and Figure 7 show in tabular and graphical form the variation of the intensities of the lines. It is quite pronounced for the hy-

TABLE V  
LINE-INTENSITY VARIATION OF  $\eta$  AQUILAE, EXPRESSED IN  
TERMS OF THE CONTINUOUS BACKGROUND INTENSITY

Plate	Phase	H $\beta$	H $\gamma$	H $\delta$	Hydrogen Mean	G	H	K
238.....	0 <sup>d</sup> 27	.246	.328	.276	.283	.143	.829	.846
217.....	0.48	.246	.257	.247	.250	.144	.852	.798
222.....	1.47	.214	.314	.220	.249	.271	.863	.889
223.....	1.47	.230	.268	.261	.253	.276	.832	.869
226.....	2.44	.244	.216	.286	.249	.250	.810	.841
229.....	3.58	.278	.308	.174	.253	.463	.856	.860
230.....	3.59	.264	.256	.243	.254	.432	.853	.882
249.....	3.93	.240	.222	.180	.214	.417	.925	.898
233.....	4.45	.265	.279	.191	.245	.403	.835	.831
254.....	4.45	.224	.306	.208	.240	.432	.827	.893
Y 620.....	4.89	.160	.171	.106	.146	.304	.....	.....
212.....	5.71	.226	.228	.207	.220	.370	.801	.834
213.....	6.69	.232	.475	.372	.360	.369	.816	.857
214.....	6.70	.351	.380	.387	.373	.300	.844	.867

drogen lines, for which the maximum strength falls near 6<sup>d</sup>7 and the minimum near 4<sup>d</sup>9, in agreement with the variation of the temperature. The variation of the intensity of the G band is also very pronounced, the minimum coinciding with the light maximum, and the maximum falling near 4<sup>d</sup>0. The amplitude of the variation of the K line is small but distinguishable. The minimum falls near the light maximum, and the maximum near 4<sup>d</sup>0. This is in general agreement with the results obtained by Yü<sup>1</sup> for RT Aurigae, S Sagittae, and T Vulpeculae. While Yü found the opposite effect for  $\eta$  Aquilae,<sup>1</sup> it

<sup>1</sup> *Ibid.*, p. 12, 1930.

was probably due to the scantiness of the available observational material. The variation of the H line is less marked and probably in part obscured by the superposition of  $H\epsilon$ , which varies in the opposite sense.

The writer is under obligation to the authorities of the Lick Observatory, who placed at his disposal the instruments and material, and loaned to him the microphotometer tracings for further study.

ST. LOUIS UNIVERSITY  
ST. LOUIS, MO.  
January 1931

## STUDIES IN PECULIAR STELLAR SPECTRA

### II. THE SPECTRUM OF B.D.-18° 3789

By W. W. MORGAN

#### ABSTRACT

The spectrum of the star B.D.-18° 3789 has been found to contain two groups of lines which periodically vary in intensity. The observations are satisfied by a period of 3<sup>d</sup>.18. The chromium lines in the spectrum form one group, while four europium lines and an unidentified line at  $\lambda$  4296 make up the other. The two groups seem always to be opposite in phase with respect to each other. Wave-lengths and identifications are given for the spectral lines. Similarities in the appearance and behavior of the spectrum to that of  $\alpha^2$  Canum Venaticorum are pointed out.

1. Variations in the relative intensities of groups of absorption lines in the spectra of variable stars are rather common phenomena, and can be explained generally as an effect of a change in temperature or pressure, or both. When, however, changes are observed in the case of a star whose light is sensibly constant, the difficulties of finding a satisfactory explanation are greatly increased. The problem becomes even more difficult when the lines of one or two elements are observed to change, while other elements having similar properties remain constant.

Probably the most remarkable examples of variations of this kind are shown by certain groups of lines in the spectra of 12  $\alpha^2$  Canum Venaticorum and 17 Leporis. There are a number of lines in the spectrum of  $\alpha^2$  Canum Venaticorum which were found by H. Ludendorff<sup>1</sup> to vary in intensity. A. Belopolsky<sup>2</sup> divided the lines into two groups which varied in a period of 5.5 days. One of these groups was identified by F. E. Baxandall<sup>3</sup> as being due to the rare earth europium, while C. C. Kiess<sup>4</sup> has provisionally identified the other with terbium. O. Struve<sup>5</sup> has found that a large number of the metallic lines in the spectrum of 17 Leporis undergo remarkable changes, both in intensity and in appearance.

<sup>1</sup> *Astronomische Nachrichten*, **173**, 1, 1906.

<sup>2</sup> *Ibid.*, **196**, 1913, and *Bulletin de l'Académie Impériale des Sciences de St. Pétersbourg* (6th ser.), **7**, 689, 1913.

<sup>3</sup> *Observatory*, **36**, 440, 1913, and *Monthly Notices*, **74**, 32, 1913.

<sup>4</sup> *Publications of the Detroit Observatory, University of Michigan*, **3**, 106, 1923.

<sup>5</sup> *Astrophysical Journal*, **72**, 343, 1930.

2. I have recently found another star which seems to belong to this rare and interesting class. The star B.D.-18°3789 (vis. mag. 5.74) is classed as of spectral type Aop by the *Henry Draper Catalogue*. The reason for the designation of "peculiar" was stated to be because of the strength of the Si II doublet,  $\lambda 4128$  and  $\lambda 4131$ .

The remarkable changes that take place in its spectrum were first observed on three spectrograms of the star which had been taken with the Bruce spectrograph in 1928. On the first of these plates, taken on March 4, 1928, the europium lines which help to make the spectrum of  $\alpha^2$  Canum Venaticorum so unusual are present, and the strongest, at  $\lambda 4205$ , is one of the most conspicuous lines in the whole spectrum. On the second plate, taken on the following night, March 5, the spectrum has about the same appearance, but on March 18, when the third plate was obtained, the entire character of the spectrum had changed in many respects. The strongest of the europium lines, which fourteen days before had been stronger than any other line in the spectrum with the exception of the hydrogen series, had faded until it was no stronger than a number of other lines in its vicinity. The other three europium lines which had been well marked on March 4 and 5 had become very faint. But the most remarkable change was the appearance of a number of lines which were either entirely absent on the earlier plates, or, in a few cases, at the limit of visibility. It was found that the lines of this second group are due to the element chromium. With a few exceptions, the lines due to other elements seem to be constant in intensity. There is a line at  $\lambda 4296.61$  which seems to vary in phase with the europium lines. The position agrees well with the Fe II line at  $\lambda 4296.56$ , but as the other iron lines in the spectrum seem to be of constant intensity, the identification with iron seems highly improbable.

3. The star was again put on the spectrographic program in January, 1931, and twelve additional one-prism plates were obtained. The variations which had been observed on the early plates were confirmed, and were found to take place in a period of a few days at most. Because of the faintness of the star and its low southern declination, it is difficult to obtain sufficiently exposed plates. It is also almost impossible to obtain more than one plate on one night. For this reason the period cannot be determined with certainty, since

periods of about 1 day cannot be excluded. The variations in intensity of the lines are, however, satisfied by a period of 3.18 days. Figure 1 shows the variation of *Cr* II 4558 and *Eu* II 4205. The phases are computed from the arbitrary epoch 1931, January 0.0. The elements of variation of *Cr* II 4558 are:

$$\text{Max} = \text{J.D. } 2426343.15 + 3^d 18E \text{ (counted from noon) .}$$

Table I gives the estimates of the intensities of the principal variable lines in the observable part of the spectrum. These estimates

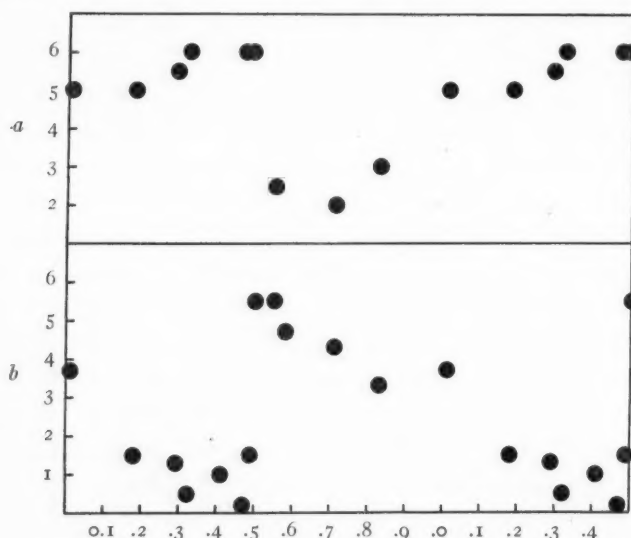


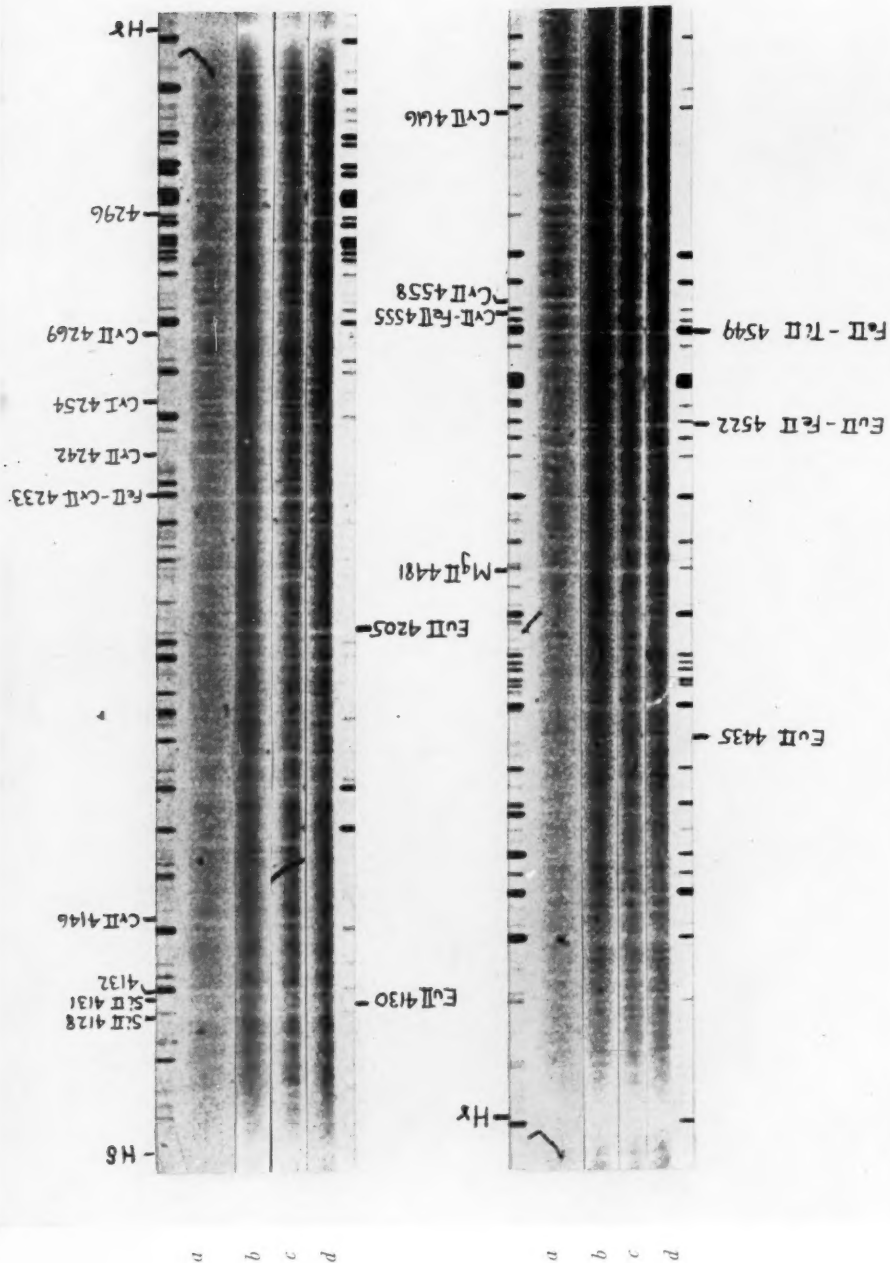
FIG. 1.—Probable curves of variation of: (a) *Eu* II 4205; (b) *Cr* II 4558. Abscissae are tenths of the probable period of 3<sup>d</sup> 18; ordinates are visual estimates of intensity.

are, as far as the upper limit of the scale is concerned, independent of the quality of the plates, as the line *Fe* II-*Ti* II 4549 has been taken as the standard for intensity 5. The lower limit of the scale is, of course, more uncertain, because of the varying quality of the plates. Only the 1931 plates have been included in Table I.

4. The range in variation of the strongest chromium lines is greater than that of the europium lines. The ultimate *Cr* I line at  $\lambda$  4254 ranges between complete invisibility and an intensity of 4, and the strongest of the *Cr* II lines at  $\lambda\lambda$  4242, 4269, and 4558, vary



# PLATE II



SPECTROGRAMS OF B.D. — 18° 37' 80"

a) 1931 Jan 14.475 (U.T.)

b) 1931 Jan 17.470 (U.T.)

c) 1928 Mar 18.381 (U.T.)

from almost complete invisibility to intensities of from 4 to 5. Plate II shows the spectrum of B.D.-18°3789. Two spectrograms are reproduced at maximum and two at minimum phases. The spectra extend from about  $\lambda 4100$  to  $\lambda 4650$ . The principal lines of chromium and europium are marked, and a few other conspicuous lines are identified. The chromium lines will be found to be stronger on the first and third reproductions than on the second and fourth. Probably the most striking illustration of the variation can be gotten by

TABLE I  
ESTIMATES OF INTENSITIES

Date U.T.	Phase	Eu II 4205	Cr II 4242	Cr I 4254	Cr II 4269	? 4296	Eu II 4435	Mg II 4481	Eu II- Fe II 4518	Cr II 4558
1931 Jan. 14.475...	0.552	2-3	3-4	3-4	3-4	1	2	6	2-3	5.5
Jan. 16.477...	.182	5	0	0	0	5	3	5	3	1.5
Jan. 17.470...	.494	6	1?	0?	0	5	2-3	3	4	1.5
Jan. 25.476...	.011	5	0?	0?	0?	5	3	3	3	3.7
Jan. 26.453...	.319	6	2	0	0	5	2-3	4	4	0.5
Jan. 30.459...	.578	.....	.....	.....	.....	2	3	4	3	4.7
Feb. 3.435...	.829	3	3	2	2	3	1	5	2-3	3.3
Feb. 5.469...	.468	6	0	0	0	5	3	1-2	4	0.2
Feb. 8.477...	.414	.....	.....	.....	.....	.....	5	.....	2-3	1
Feb. 9.426...	.713	2	2-3	3-4	1?	1:	1	6	2	4.3
Feb. 14.431...	.287	5-6	.....	.....	.....	4	2	2-3	3-4	1.3
Feb. 21.476...	0.502	.....	.....	.....	.....	.....	6	.....	1?	5.5

The phases are expressed in decimal fractions of the period. Complete absence of a line is denoted by intensity zero.

comparing the *Fe II-Ti II* line at  $\lambda 4549$  with the two chromium lines at  $\lambda 4555$  and  $\lambda 4558$ . The europium lines vary in the opposite sense. The most marked variation among the europium lines is shown by  $\lambda 4205$ , which is stronger on the second and fourth spectrograms. Only a few of the variable lines have been marked. It is possible that the hydrogen lines and *Mg II* 4481 also vary. Over thirty *Cr II* lines were measured, including a number predicted by T. Dunham and Miss Charlotte E. Moore<sup>1</sup> from a study of incomplete multiplets and found by them in  $\alpha$  Persei and in the sun. Owing to the fact that B.D.-18°3789 is at a higher temperature than  $\alpha$  Persei, many of the arc lines which cause blends with chromium in the cooler star have disappeared, and most of the *Cr II* lines are practically un-

<sup>1</sup> *Ibid.*, 68, 37, 1928.

blended. The star is therefore well suited to the study of the relative intensities of the chromium lines which have not been observed in the laboratory.

Table II lists the *Cr* II lines observed in the star between the limits  $\lambda\lambda$  4030–4635. The columns give: (1) the laboratory wave-lengths for all lines observed in the laboratory and the computed wave-lengths for the lines predicted by Dunham and Miss Moore; (2) the intensities in arc and spark, according to Exner and Haschek; (3) the multiplet number of the lines. The multiplets have been consecutively numbered, from lower to higher wave-length. The final columns in the table give: (4) the wave-lengths in B.D.-18°3789; (5) the maximum and minimum intensities in the star; and (6) notes as to blends, etc.

Dunham states<sup>1</sup> that the strongest lines of neutral and singly ionized chromium are of about equal intensity in  $\alpha$  Persei. As the strongest *Cr* I line, located at  $\lambda$  4254, is approximately equal to the strongest enhanced lines in B.D.-18°3789, we would expect the spectral type of the two stars to be about the same, but in other respects they are quite different.

5. The lines in the spectrum of europium have recently been classified according to temperature by A. S. King,<sup>2</sup> who finds four lines within my observed range which are so strong in the arc that they dominate the spectrum. They are probably lines of low energy-level in the spectrum of *Eu* II, and are strongly reversed in the arc. Table IV gives the wave-lengths of these lines, their intensities in the electric furnace and the arc, the wave-lengths measured in the star, and the estimated intensities of the lines at their greatest and least strength in the star. The estimates are on the same scale as in the preceding tables.

The line at  $\lambda$  4129.78 is situated between the two *Si* II lines at  $\lambda$  4128 and  $\lambda$  4131. It is probably entirely unblended. It seems likely that this line, together with  $\lambda$  4132, causes the star to seem to have strong *Si* II lines on objective-prism plates. The silicon doublet itself is of only moderate intensity, and seems hardly strong enough to designate the spectrum as "peculiar." In the cooler stars there is a

<sup>1</sup> *Contributions from the Princeton University Observatory*, No. 9, 7, 1929.

<sup>2</sup> *Astrophysical Journal*, 72, 221, 1930.

TABLE II  
LINES OF *Cr* II

$\lambda$	Int.	Mult	$\lambda$ Star	Max. Int.	Min. Int.	Notes
4030.37....	Pred.	1	4030.48	3:	0	
4053.44....	Pred.	2				
4054.10....	Pred.	2	4054.05	3-4	1	
4064.05....	Pred.	1	4064.00	3-4	0?	Blended with <i>Fe</i> I 4063.61; <i>Cr</i> II probably disappears at minimum
4070.94....	0-4		4070.84			
4072.63....	Pred.	3	4072.74	2	0	
4075.66....	Pred.	2	4075.73	2	1	
4076.87....	Pred.	2	4076.93	3	0	
4077.55....	Pred.	2	4077.61	4	1-2	<i>Sr</i> II and <i>Cr</i> II contribute about equally at maximum
4087.63....	Pred.	2	4087.47	1	0	
4088.85....	Pred.	2				
4111.09....	0-4		4110.98	3	0	
4112.57....	Pred.	4				
4145.78....	0-6					
4146.45....	Pred.	3	4146.00	4	0	Enhanced line listed by Lockyer
4170.64....	Pred.	4	4170.79	2	1	
4171.92....	Pred.	4	4171.95	3-4	1	
4172.62....	Pred.	4				
4179.41....	0-3	3	4179.22	4	0?	
4207.34....	Pred.	3	4207.45	1	1	Line may not be due to <i>Cr</i> II
4215.78....	Pred.	4	4215.60	3-4	1-2	<i>Sr</i> II and <i>Cr</i> II contribute about equally at maximum
4217.08....	Pred.	4	4217.38	3	3	<i>Cr</i> II probably contributes little, if at all, to the star line. In Lockyer's list of enhanced lines; not mentioned by Dun- ham
4224.86....	0-2		4224.58	3-4	1	
4229.82....	Pred.	3	4229.77	2	2	The contribution of <i>Cr</i> II is prob- ably negligible
4233.28....	Pred.	5	4233.25	8	5	<i>Cr</i> II and <i>Fe</i> II about equal in in- tensity at maximum
4242.37....	1-5	5	4242.48	3-4	1	
4252.65....	0-1	5	4252.61	3	0	
4261.81....	0-2	5	4261.94	3	2-3	{Line apparently not variable Probably a blend
4261.90....						
4269.30....	0-1	5	4269.11	3-4	0	
4275.58....	0-1	5	4275.32	3	1-2	
4279.00....	Pred.	6				
4280.34....	Pred.	6	4280.83	1-2	2-3	Line probably not due to <i>Cr</i> II
4284.20....	0-2	5	4284.20	2	0	
4504.56....	Pred.	7				
4507.22....	Pred.	7	(4507.74)	1	1:	Blended with <i>Fe</i> II 4508.29
4542.82....	Pred.	7				
4544.69....	Pred.	7				
4555.09....	1-3	8	4555.21	4n	2	Blended with <i>Fe</i> II 4555.89
4558.66....	1-20	8	4558.71	5-6	1:	
4558.84....	Pred.	8				

TABLE II—Continued

$\lambda$	Int.	Mult	$\lambda$ Star	Max. Int.	Min. Int.	Notes
4571.30....	Pred.	7	4572.20	2	2	Line probably entirely due to <i>Ti</i> II 4571.98
4572.83....	Pred.	7				
4588.21....	1-15	8	4588.10	2-3	1	
4588.40....	Pred.	7				
4589.89....	Pred.	8	4590.07	2	1:	
4592.06....	0-2	8	4592.07	2	1:	
4616.72....	0-3-4	8	4616.37	3	1:	
4618.82....	0-8	8	4619.00	2	0	
4621.51....	Pred.	9				
4634.12....	0-8	8	4634.18	2n	0	

TABLE III

LINES OF *Cr* I

$\lambda$	Int.	Mult	$\lambda$ Star	Max. Int.	Min. Int.	Notes
4254.348...	50R-50	1	4254.39	3-4	0	Partially masked by 4275.32? Blended with <i>Ti</i> II 4290.23
4274.808...	50R-30	1	.....	1-2?	0	
4289.731...	30R-30	1	4289.89	4	3-4	

TABLE IV

LINES OF EUROPIUM

$\lambda$ (KING)	INT.		$\lambda$ (STAR)	INT.	
	Fur.	Arc		Max.	Min.
4129.639.....	4?	25?	4129.78	2-3	2:
4129.734.....	80	500R			
4204.909.....	5?	30?			
4205.046.....	100	600R	4205.04	7	3-4
4435.473.....	10	20?			
4435.602.....	80	400r	4435.49	3-4	1:
4522.602.....	50	200r	4522.56	4-5	2-3

rather strong line of ionized yttrium at  $\lambda$  4204.69, but apart from the disagreement in wave-length with the *Eu* II line at  $\lambda$  4205.05, the star does not seem to contain any yttrium lines. There is also a line due to *V* II at  $\lambda$  4205.09, but, as stronger lines due to the same element seem to be absent, it can make no sensible contribution to the intensity of the well-marked star line. There is a strong neutral cal-

cium doublet at the position of *Eu* II 4435, but as the ultimate line of *Ca* I at  $\lambda$  4226 is either absent or very weak in the star, it does not seem that calcium can sensibly disturb the europium line. *Eu* II 4522.56 is blended with the *Fe* II line at  $\lambda$  4522.64. The two elements probably contribute about equally to the intensity of the stellar line.

6. The principal lines in the spectrum, with the exception of the hydrogen series, were measured for wave-length. The measures were made when the chromium lines were strong. Table V gives the wave-lengths and identifications for the strongest star lines between the limits  $\lambda$  4030 and  $\lambda$  4634. The columns give: (1) the measured wave-length; (2) the intensity of the stellar line; (3) the identification and wave-length of the line in the laboratory. In the cases of serious blends the method of Dunham has been used to show the approximate share of the different elements in the blend. Thus, if two elements contribute equally, each is preceded by one plus sign (+). If one contributes twice as much as the other, it is preceded by two plus signs (++), while the other is preceded by one plus sign (+). The approximate share of the different elements in the case of blends was determined from the intensities of unblended members of the same multiplets to which the elements involved belong. A few notes on peculiarities of individual lines are placed at the end of the table.

7. The plates were all measured for radial velocity. The results are shown in Table VI. The columns give (1) the date on which the plate was obtained; (2) the radial velocity; (3) the quality of the plate; and (4) the measurer of the radial velocity.

In spite of the poor quality of most of the plates, the scatter in the measures is larger than it should be, if the velocity is constant. The two best plates of the 1931 series give positive velocities, while the three 1928 plates give negative velocities of from 14 to 23 km. The observations in 1931 were plotted according to the phases computed from the variable chromium line at  $\lambda$  4558. No certain periodicity was shown. It seems probable that the radial velocity is variable, but a long series of good plates will be necessary to settle the question.

Adams and Joy<sup>1</sup> have published a value of  $-9.5 \pm 1.8$  km/sec. for the radial velocity. This value was determined from five plates. The

<sup>1</sup> *Ibid.*, 57, 149, 1923.

mean velocity from the Yerkes measures is  $-8.2$  km/sec., in very good agreement with the Mount Wilson value.

TABLE V  
WAVE-LENGTHS AND IDENTIFICATION OF LINES

$\lambda$	Int.	Identification	$\lambda$	Int.	Identification
4046.02.....	2-3	Fe I 45.82	4289.89.....	4	Ti II 90.23
4049.07.....	3	-?Cr I 48.76	4294.23.....	1	+Fe I .13; +++Ti II .10
4054.05.....	3-4	+Cr II .08; +Ti II 53.84	4296.61.....	1	Fe II .56
4064.00.....	4	+++Cr II .05; +Fe I .60	4299.87.....	2	Ti II 00.05
4070.84.....	2	Cr II .99	4301.97.....	2	Ti II .93
4072.74.....	3	Cr II .63	4303.19.....	3	Fe II .18
4075.73.....	2	Cr II .66	4307.80.....	1-2	?Ti II .89
4076.93.....	3	Cr II .88	4312.87.....	1-2	Ti II .88
4077.61.....	4	+Sr II .71; +Cr II .58	4314.96.....	3	Ti II .98
			4325.61.....	4	-Fe I .77
4113.26.....	3-4	.....	4330.53.....	3	+Ti II .26; +Ti II .71
4119.23.....	1-2N	.....	4351.82.....	1	Fe II .77
4122.98.....	3-4N	-?Fe II .67	4367.97.....	2-3	Ti II .67
4128.02.....	3-4	Si II .05	4374.72.....	2	Sc II .46
4129.91.....	2	Eu II .73	4383.57.....	2	Fe I .55
4131.02.....	1-2	Si II 30.88	4385.17.....	2-3	Fe II .39
4132.37.....	4	?O .54; ?Fe I .07	4395.27.....	2	Ti II .04
4138.10.....	1-2N	.....	4400.01.....	2	Ti II 99.77
4143.50.....	2-3	-?Fe I .88	4404.99.....	1	Fe I .75
4146.00.....	4	Cr II 45.78	4410.85.....	1	Ti II 11.10
4161.40.....	4	-?Ti II .52 ch	4415.00.....	1	Fe I .13
4163.85.....	2-3	Ti II .65 ch	4416.82.....	2	Fe II .81
4170.79.....	2	Cr II 64	4435.44.....	1	Eu II .60
4171.95.....	3-4	+ +Cr II .92; +Ti II .90	4444.23.....	2-3N	?Ti II 43.80
4173.53.....	2-3	Fe II .48	4481.31.....	5	Me II .25
4179.22.....	4	+ +Cr II .41; +Fe II .78.87	4489.02.....	1-2N	Fe II .21
			4501.02.....	2N	Ti II .27
4181.85.....	2	?Fe I .77	4507.81.....	1	+Cr II .22; +Fe II .08.29
4186.92.....	3	?Fe I 87.05			?Cr I .91
4190.28.....	2	-?Ti II .29	4511.74.....	2	+Fe II .34 +?
4191.37.....	2	?Fe I .44	4515.49.....	3-4	Fe II .24
4195.46.....	2	?Fe I .34	4520.16.....	2	+Fe II .64; +Eu II .60
4198.43.....	3	?Fe I .31	4522.60.....	2-3	Ti II .97
4205.10.....	3-4	Eu II .05	4533.97.....	1-2	.....
4207.45.....	1	Cr II .34	4535.52.....	1-2	.....
4215.60.....	3-4	+Cr II 78; +Sr II .52	4540.57.....	4NN	.....
4217.38.....	3	Cr II .08	4545.68.....	2NN	.....
4224.58.....	3	?Cr II .86	4549.58.....	5	+ + +Ti; .64 +Fe .48
4227.49.....	3	-?Fe I .44	4555.21.....	4N	Cr II .09
4229.77.....	2	Cr II .82	4558.71.....	5	+ + +Cr II .66; ?+Cr II .84
4233.25.....	7	+Cr II .28; Fe II 16			Ti II .77
4242.48.....	3-4	Cr II .37	4563.93.....	1	?Cr I .51
4246.66.....	3	Sc II .83	4565.76.....	1	-Cr II 71.30; + + +Ti 71.98; -Cr II 72.83
4250.40.....	3N	+Fe I .13; + + +Fe I .79	4572.20.....	2-3	Fe II .31
4252.61.....	3	Cr II .65	4576.53.....	1-2	+Fe II 2.84; +Fe II 3.84 + +?
4254.39.....	3-4	Cr I .34	4583.11.....	3NN	+ + +Cr II .21; +Cr II .41
4258.00.....	3	?Fe II .17			+ [Cr II .89; Cr II 95]
4261.94.....	3	+Cr II .81; + + +Cr II .90	4588.10.....	3	+ + Ti II .96
4263.96.....	3NN	.....	4590.07.....	2	Cr II .06
4269.11.....	3-4	Cr II .30			Cr II .72
4271.56.....	2	Fe I .76	4592.07.....	2	Cr II 18.82
4273.35.....	2	Fe II .33	4616.37.....	3	+?Fe I .05; +?Cr I 26.18
4275.32.....	3	Cr II .58	4619.00.....	2	Fe II .33
4278.04.....	2-3	Fe II .17	4625.55.....	2-3	Cr II .12
4280.83.....	1	+Cr II .34; + + +Cr II 81.08	4629.22.....	2	
4284.20.....	2	Cr II .20	4634.18.....	2N	

The line at  $\lambda$  4113.26 seems to vary in phase with the Cr II lines. Exner and Haschek list a chromium line of intensity 1 in the spark at  $\lambda$  4113.18. The line at 4444.23 gave discordant velocities on several of the plates. It is possible that it varies in position.

8. If the variation in intensity of the *Cr* II lines be considered as due to periodic changes in effective excitation in the atmosphere of the star, difficulties immediately arise. On the assumption of changes in excitation, neutral chromium should vary oppositely in phase to ionized chromium, since the increased strength of *Cr* II would be obtained at the expense of *Cr* I. (From the general appearance of the spectrum, it would seem that the higher stages of ionization can be

TABLE VI  
MEASURES OF RADIAL VELOCITY

Date U.T.	Vel.	Qual.	Meas.
1928 Mar. 4.438....	-20.4	g	$\sigma$
	-23.3		M
Mar. 5.342....	-17.9	g	$\sigma$
	-17.9		M
Mar. 18.381....	-13.7	g	$\sigma$
	-16.0		M
1931 Jan. 14.475....	-15.1	f	M
Jan. 16.477....	-16.2	p	M
Jan. 17.470....	+ 5.1	g	M
Jan. 25.476....	- 9.2	p	M
Jan. 26.453....	+ 2.4	g	M
Jan. 30.459....	- 7.9	vp	M
Feb. 3.435....	- 9.7	p	M
Feb. 5.469....	+ 7.2	f-p	M
Feb. 8.477....	- 3.8	p	M
Feb. 9.426....	- 0.8	f	M
Feb. 14.431....	- 7.0	p	M
Feb. 21.476....	-13.7	p	M

$\sigma$ =O. Struve; M=W. W. Morgan.

neglected.) The observations, however, show that the line of *Cr* I at  $\lambda 4254$  varies in phase with *Cr* II. The other two members of the ultimate triplet of *Cr* I do not show much variation, probably because of blends. It thus seems that, as far as can be judged by appearances, the actual number of chromium atoms effective in absorbing the observed lines varies in the period of the variation in intensity. In the case of the europium lines, there is no check on the behavior of the lines of the neutral atom. It is difficult to find any relationship between the elements chromium and europium, but their complementary behavior seems very suggestive.

It also seems remarkable that, in the only other star known to contain strong lines of europium, the same lines form a group which

varies in intensity. In  $\alpha^2$  Canum Venaticorum the europium lines vary from almost complete invisibility to intensities which place them among the strongest lines in that star's spectrum. It is not possible to generalize with only two stars, but it seems possible that europium is connected with some state of physical change. It would be necessary to find more of such stars before an adequate study could be made of the phenomenon. The importance of such a study can hardly be overemphasized, as it seems that in stars of this class we are observing results of changes which may illustrate the basic laws of stellar evolution.

YERKES OBSERVATORY  
WILLIAMS BAY, WIS.  
February 16, 1931

## APPARENT VELOCITY-SHIFTS IN THE SPECTRA OF FAINT NEBULAE<sup>1</sup>

By MILTON L. HUMASON

### ABSTRACT

Apparent velocity-shifts of the spectral lines of 46 extra-galactic nebulae have been observed at Mount Wilson, 9 of them by F. G. Pease. Most of these objects are fainter and more distant than any heretofore observed; approximately half of them are cluster nebulae. With one exception these observations confirm Hubble's velocity-distance correlation and provide numerical data which may be used in discussions of the significance of the red-shift of spectral lines.

For all faint nebulae the displacements are large and toward the red, the maximum of the continuous spectrum shifting also.

The largest apparent velocity-shift observed,  $+19,700$  km/sec., is that of the brightest nebula in W. H. Christie's cluster in Leo. Its photographic magnitude is 16.8.

With the exception of N.G.C. 205, classified as F5, the spectral types of all the nebulae having absorption lines fall within the narrow limits G1-G5.

In 1929 Hubble found a relation connecting the velocities and distances of the extra-galactic nebulae for which spectra were then available.<sup>2</sup> The spectra were, in general, those of the nearer and brighter nebulae, and the relation was thus established out to the nearest of the great clusters of nebulae—the Virgo cluster at a distance of the order of two million parsecs. A program of investigation was immediately planned with a view to testing the validity of the relation over as great a range in distance as could be covered with the 100-inch reflector. Spectra of 46 of the fainter nebulae have now been observed. With one exception, possibly the velocity of an isolated object seen in projection on a remote cluster, the new data fully confirm the velocity-distance relation<sup>3</sup> previously formulated and extend the observational range to a distance of about thirty-two million parsecs. This phase of the investigation will be presented in a joint paper by Hubble and myself. The present discussion deals primarily with the spectra themselves.

<sup>1</sup> *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 426.

<sup>2</sup> *Mt. Wilson Communications*, No. 105; *Proceedings of the National Academy of Sciences*, 15, 168, 1929.

<sup>3</sup> It is not at all certain that the large red-shifts observed in the spectra are to be interpreted as a Doppler effect, but for convenience they are expressed in terms of velocity and referred to as apparent velocities.

## THE SPECTROGRAPH

In general the spectra were photographed at the Cassegrain focus of the 100-inch reflector. Nebulae very rarely present uniform surfaces, and for the smaller condensations at least, the large aperture of the 100-inch is more efficient photographically than that of the 60-inch reflector. Moreover, by selecting the condensed elliptical nebulae with highly concentrated semistellar nuclei, it was possible to obtain with the 100-inch practically the full advantage which it offers in the case of stars.<sup>1</sup>

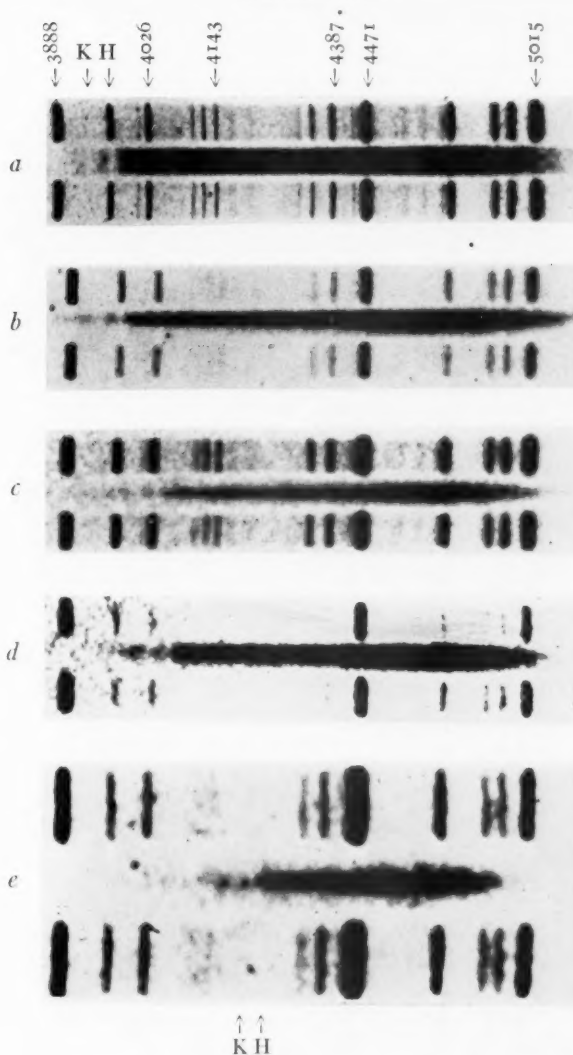
The first spectrograph used was provided with a 24-inch collimator, two prisms of light flint glass, and a 3-inch camera giving a dispersion of 170 Å per millimeter at  $\lambda$  4350. This combination proved too slow for efficient observation of faint nebulae. With prolonged exposures it was possible to record the region  $\lambda\lambda$  4200–5000; in general, however, the G band ( $\lambda$  4303) was the only feature strong enough to be seen clearly, but, on account of the large displacements encountered, the identification was very uncertain.<sup>2</sup> For this reason one prism was removed, thus giving a dispersion of about 340 Å per millimeter at  $\lambda$  4350. The increased speed made it possible to register the spectra of faint nebulae to the violet of  $\lambda$  4000 and hence to use the unmistakable H and K lines as a basis for identification of the other features.

The reduction in scale proved so effective that an even smaller dispersion seemed desirable. This was made possible by Dr. W. B. Rayton, of the Bausch and Lomb Optical Company, who designed a spectrograph objective having a ratio of F/0.6. The Rayton lens with two prisms gives a dispersion of about 418 Å per millimeter at

<sup>1</sup> Hubble, *Mt. Wilson Contr.*, No. 398; *Astrophysical Journal*, **71**, 231, 1930. The steepness of the luminosity gradient in elliptical nebulae is illustrated by Fig. 1a, which gives transparency-curves for photographic images of N.G.C. 3379. The semistellar nucleus is indicated by the sharp maxima shown by the two shortest exposures.

<sup>2</sup> The case of N.G.C. 4884 is an example. The first spectrum obtained with the 60-inch reflector was very weak. The stronger of the two lines measured was assumed to be  $\lambda$  4383 Fe, and gave the velocity +1500 km/sec. announced in *Summary of the Year's Work at Mount Wilson* for 1928. This seemed the most probable identification, since the highest velocity then known was V. M. Slipher's value, +1800 km/sec. for N.G.C. 584. A later spectrogram, showing the H and K lines of calcium, proved the true red-shift to be +6700 km/sec., and the strong line originally measured to be the G band.

# PLATE III



Nebular spectra showing increase in red-shift with decreasing apparent brightness corresponding to increasing distance. Arrows indicate positions of H and K in (a) and (e). For (a) to (d) the enlargement is 28 times the scale of the original negatives; for (e), photographed with about one-half the dispersion used for the others, 47 times.

- a) Sky; normal position of H and K; wave-lengths of several of the helium comparison lines are indicated.
- b) N.G.C. 221 (M 32); apparent velocity,  $-185$  km/sec.
- c) N.G.C. 385; apparent velocity,  $+4900$  km/sec.
- d) N.G.C. 4884; apparent velocity,  $+6700$  km/sec.
- e) Brightest nebula in Leo cluster; apparent velocity,  $+19,700$  km/sec.



$\lambda 4500$ , and with one prism, about 875 Å per millimeter for the same region. This is much the fastest combination in actual use at Mount Wilson.<sup>1</sup> The definition is excellent, and four lines are generally recognizable in nebular spectra. These lines are H and K,  $H\delta$  ( $\lambda 4101$ ), and the G band ( $\lambda 4303.14$ ). Occasionally  $H\gamma$  ( $\lambda 4340$ ),  $\lambda 4383$  Fe, and  $H\beta$  ( $\lambda 4861$ ) can also be identified.

#### OBSERVING PROGRAM

The nebulae selected for observation are about equally divided between clusters and isolated objects. Clusters of nebulae offer the great advantage that fairly reliable distances can be derived from the mean luminosities of the many individual members, while the observations can be restricted to the several brightest members of each cluster. Thus the greatest possible distance for a given apparent luminosity is assured. Further, since there appears to be no correlation of red-shift with absolute luminosity among nebulae whose distances are known, the several brightest members of a cluster may safely be assumed to represent the cluster as a whole in respect to line displacement.

The isolated nebulae were included in order to test the possibility of a systematic difference between them and the cluster nebulae, and later to afford data for special problems involving the distances. Since apparent luminosity furnishes only a statistical criterion of distance, it was necessary to observe enough isolated objects to form several groups. Mean velocities could then be compared with the mean distances of the groups.

Table I lists the nebulae observed, together with the measured apparent velocities, spectral types, and estimated uncertainties. The uncertainties are possibly three times the probable errors as formally derived from the few lines measured and are believed to be a fair indication of the reliability. They depend on the scale, the exposure, and the number of lines that could be measured.

Spectra of 9 of the 46 nebulae were photographed by F. G. Pease, who has kindly placed the spectrograms at my disposal for

<sup>1</sup> For a further description of the extraordinarily efficient Rayton lens, together with some account of its performance, see Rayton, *Astrophysical Journal*, **72**, 59, 1930, and Humason, *Mt. Wilson Contr.*, No. 400; *Astrophysical Journal*, **71**, 351, 1930.

TABLE I  
APPARENT VELOCITY-SHIFT AND SPECTRAL TYPE

N.G.C.	Apparent Velocity-Shift	Estimated Uncertainty	Spectral Type	Remarks
	km/sec.	km/sec.		
380.....	+ 4400	75	G5	Group in Pisces; not one of the large clusters but a group of about 25 nebulae
383.....	+ 4500	100	G3	
384.....	+ 4500	100	G5	
385.....	+ 4900	100	G5	
1270.....	+ 4800	100	G4	Cluster in Perseus
1273.....	+ 5800	75	G5	
1275.....	+ 5200	25	G+P (pec.)	
1277.....	+ 5200	75	G3	
2562.....	+ 5100	100	G4	Cluster in Cancer
2563.....	+ 4800	100	G4	
*.....	+ 19700	300†	G5	Christie's cluster in Leo
‡.....	+ 11700§	200	G5	Baade's cluster in Ursa Major
4192  .....	+ 1150§	100	G2	Cluster in Virgo
4374  .....	+ 1050	100	G4	
4853  .....	+ 7600§	100	G1	Cluster in Coma Berenices; N.G.C. 4865 may not be a member; velocity of N.G.C. 4884 previously announced as +1500 km/sec.
4860  .....	+ 7900§	75	G3	
4865.....	+ 5000§	75	G3	
4872.....	+ 6900	200†	G3	
4874.....	+ 7000	200†	G4	
4881.....	+ 6900	200†	G3	
4884.....	+ 6700§	75	G3	
4895.....	+ 8500	200†	G4	
I.C. 4045.....	+ 6600	200†	G1	
7611.....	+ 3400	75	G2	Cluster in Pegasus
7617.....	+ 3900	100	G1	
7619.....	+ 3800§	75	G3	
7623.....	+ 3800	125	G2	
7626  .....	+ 3700	100	G3	
205.....	- 300§	50	F5	Distant companion of An- dromeda
2859.....	+ 1500	100	G3	
2950.....	+ 1500	75	G4	
3193.....	+ 1300	100	G3	
3227.....	+ 1150§	30	G+Pd	
3610.....	+ 1850	75	G2	
4051.....	+ 650§	40	G+Pb	
5457.....	+ 300¶	25	G+Pd	

\* The nebula observed is the brightest one in the cluster.

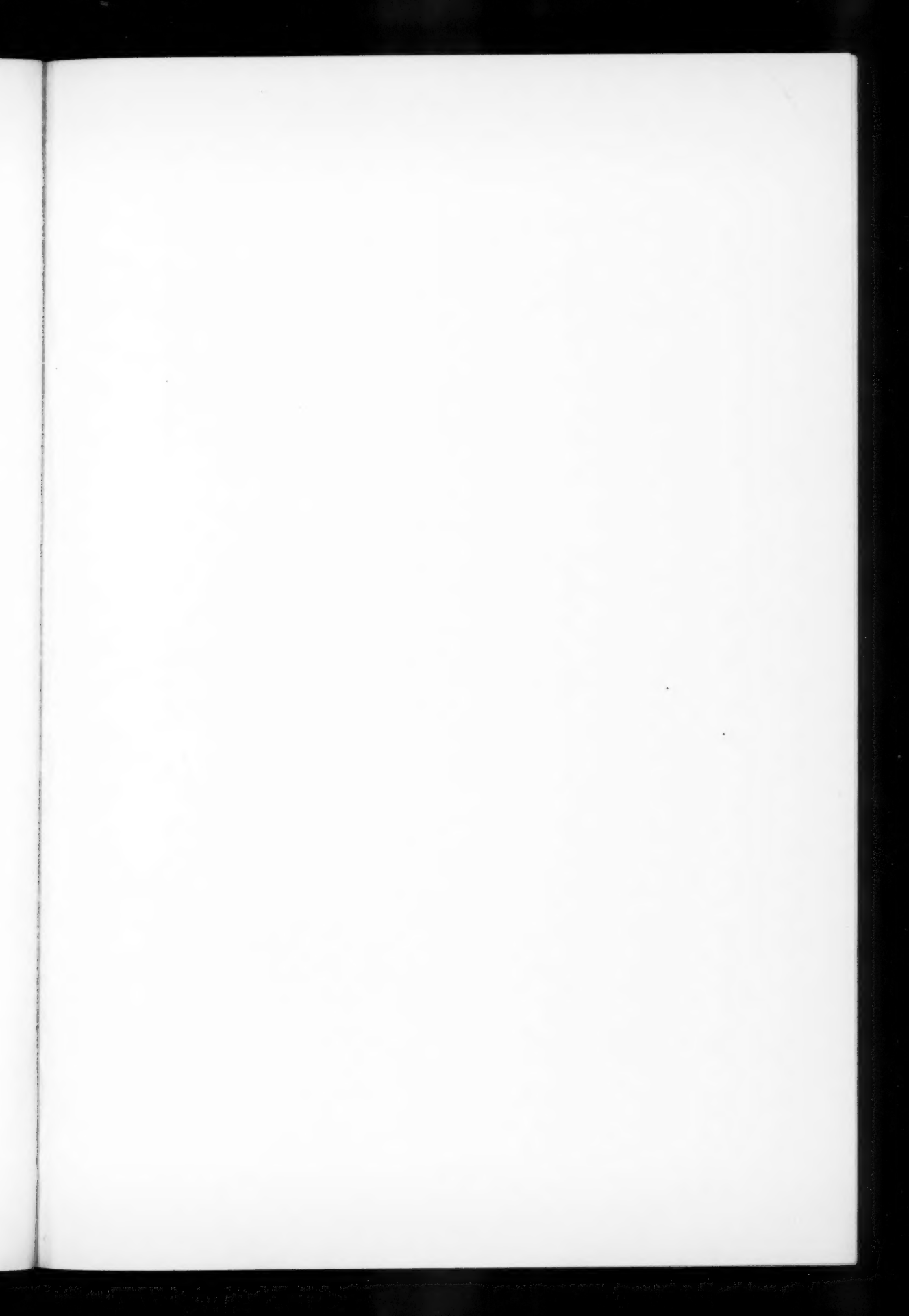
† The large uncertainty is due to the dispersion, 875 Å per millimeter.

‡ The velocity is from the brightest nebula, Baade No. 24.

§ Velocity previously announced.

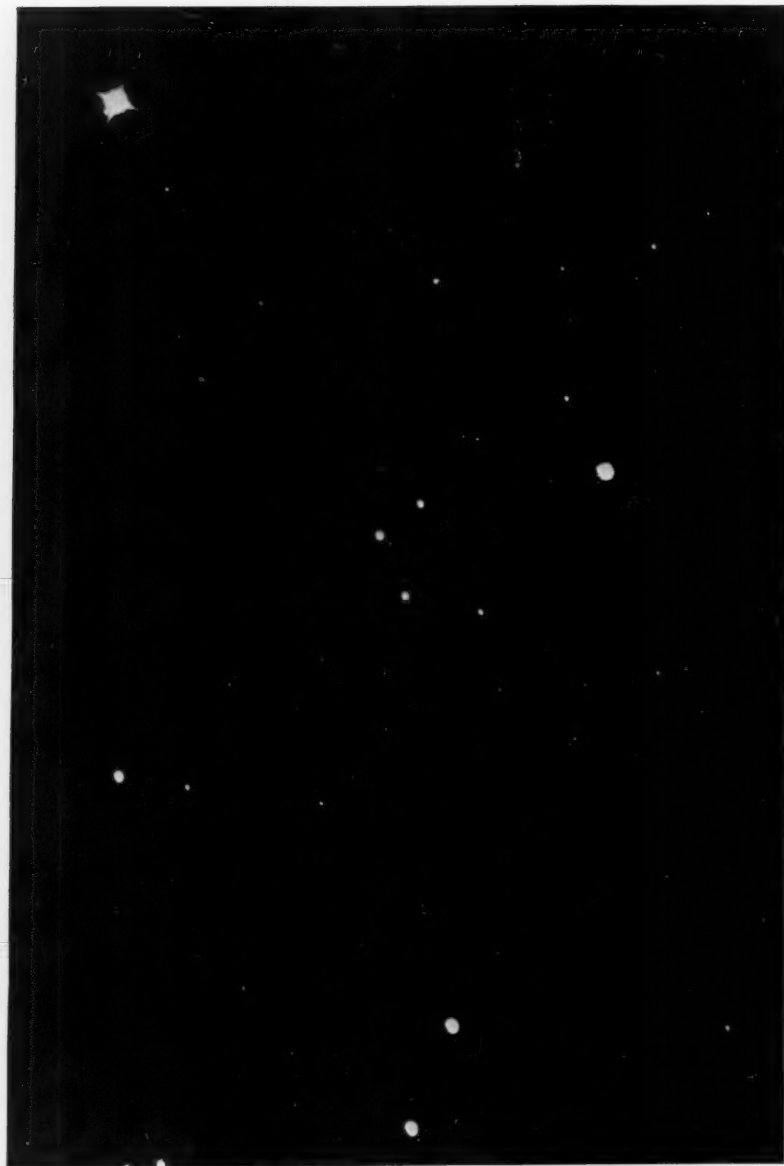
|| Spectrum obtained by F. G. Pease.

¶ The velocity previously published was +200 km/sec. Recently the nucleus and two emission patches have been observed, making the mean +300 km/sec.



# PLATE IV

N



W

Small region in the Leo cluster; R.A.  $10^h 24^m 11^s$ , Dec.  $+10^\circ 47'$  (1930). The nebula observed, the brightest in the cluster, photographic magnitude about 16.8, is indicated by the arrow. The bright star in the upper right-hand corner is B.D.  $+11^\circ 2230$ . Scale, 1 mm =  $7''$ .

TABLE I—*Continued*

N.G.C.	Apparent Velocity-Shift	Estimated Uncertainty	Spectral Type	Remarks
	km/sec.	km/sec.		
6359.....	+ 3000§	75	G3	
6658.....	+ 4100	75	G3	
6661.....	+ 3900	100	G5	
6702  .....	+ 2250	75	G4	
6703  .....	+ 2000	75	G3	
6710.....	+ 5100	100	G3	
6822.....	- 150§	25	Pd	Emission patch in N.G.C. 6822
6824.....	+ 3200	75	G4	
7217  .....	+ 1050	100	G4	
7242  .....	+ 5000	200	G3	The large uncertainty is on account of a weak plate

measurement and discussion. Velocities of 13 of the nebulae have previously been published, 4 of them by Pease and myself jointly. The table thus includes all recent measures made at Mount Wilson. The velocities previously available, for the most part those made by V. M. Slipher at the Lowell Observatory, but including 7 measures from Mount Wilson and 3 from Lick Observatory, have been collected by G. Strömberg.<sup>1</sup> The two lists together include all velocities published to date.<sup>2</sup>

The present list gives velocities of 24 nebulae in 7 clusters, 4 in a group in Pisces, and 18 isolated objects. The nebulae range from irregular objects (N.G.C. 6822) and late-type spirals (M 101) to the early elliptical nebulae which predominate in the clusters. The largest displacement found is that for the brightest nebula in Christie's cluster in Leo,<sup>3</sup> which has a photographic magnitude of about 16.8. The observation was made with the smallest dispersion used, but the measured displacement of +19,700 km/sec. is believed correct within a few hundred kilometers. The single spectrogram available was exposed 13 hours and is one of the best so far obtained.

Where several velocities have been measured in a single cluster, the range is small compared with the mean in all except the Coma

<sup>1</sup> *Mt. Wilson Contr.*, No. 292, p. 2; *Astrophysical Journal*, 61, 354, 1925.

<sup>2</sup> Slipher reports spectra for three additional nebulae in the Virgo cluster, but has published no velocities as yet.

<sup>3</sup> This cluster was found by W. H. Christie, on plates taken at Mount Wilson with the 60-inch reflector. The 1930 position is: R.A. 10<sup>h</sup>24<sup>m</sup>11, Dec. +10°47'.

cluster. There, around a mean of  $+7300$  km/sec., 8 nebulae show a range of  $1900$  km/sec., which is the maximum among all the clusters. A ninth object (N.G.C. 4865) gives  $+5000$  km/sec. On direct photographs this object is in no way distinguishable from the other bright cluster nebulae, but the velocity derived from two spectrograms is the one conspicuously discordant result in the table.

#### MEASUREMENTS

On the average, the velocities depend on measures of about three lines. These are generally H and K and the G band ( $\lambda 4303$ ), with occasionally one or more of the lines  $H\delta$  ( $\lambda 4101$ ),  $H\gamma$  ( $\lambda 4340$ ),  $\lambda 4384$  Fe, and  $H\beta$  ( $\lambda 4861$ ), according to the density of the spectrograms in the region of the lines. As an example of the results obtained from spectra having a dispersion of  $875$  Å per millimeter, the individual measurements are listed for the brightest nebula in

	E.M.	M.H.
3933 (K).....	$+19,890$ km/sec.	$+19,925$ km/sec.
3968 (H).....	$19,571$	$19,708$
4101 ( $H\delta$ ).....	$19,609$	$19,615$
4303 (G).....	$19,778$	$19,276$
4340 ( $H\gamma$ ).....	$+19,815$	$+19,579$
Mean.....	$+19,733$	$+19,621$

Christie's cluster. In Table I the probable uncertainty is given as  $300$  km/sec., and the apparent velocity entered to the nearest  $100$  km/sec.

The wave-length of the blend which forms the G band ( $\lambda 4303.14$ ) was derived from spectra of standard velocity stars having as nearly as possible the same type as the nebulae. Before this wave-length was obtained, the value  $\lambda 4307.91$  was used in reducing measures of N.G.C. 4853, 4860, and 4865, but their velocities have since been corrected by an average of  $+233$  km/sec.

Each spectrogram has been measured twice, once by Miss Elizabeth MacCormack and once by the writer. These duplicate measures are in good agreement. Velocities less than  $2000$  km/sec. are given to the nearest  $50$  km/sec., those larger to the nearest  $100$  km/sec.

Valuable confirmation that the velocity displacements,  $\delta\lambda/\lambda$ , do

not vary appreciably with the wave-length is afforded by the emission spectra from the nucleus of N.G.C. 1275 in the Perseus cluster of nebulae. The measured lines range from  $\lambda$  3727 to  $\lambda$  4861, with no systematic difference in the velocities.

#### DETERMINATION OF SPECTRAL TYPES

Except for the bright-line nebulae, classifications have been based on a comparison with spectra of N.G.C. 221 (M 32). This nebula was chosen as the standard type because plates of high dispersion were available, from which the spectrum was classified as dG<sub>3</sub> by Adams, Joy, and the writer. Additional spectrograms were obtained with the different dispersions in order that comparisons might always be made between spectra having the same scale.

The criteria used in assigning types to the bright-line nebulae are those adopted by the International Astronomical Union and given in the *International Critical Tables*.

Nebulae having absorption lines show only a small dispersion in type, all of them, except N.G.C. 205, falling within the narrow limits G1-G5. N.G.C. 205 has been classified as F5.

Bright-line spectra are of two different types, according as the continuous spectrum is strong or weak in comparison with the emission. The relative intensity of the continuous spectrum in the first type appears the same as in spectra having absorption lines. Examples are N.G.C. 3227, 4051, and 1275—all nebulae having bright stellar nuclei. The emission lines in the spectrum of N.G.C. 1275 are shifted to the red by the same amount as the absorption lines in the spectra of other members of the cluster to which it belongs, namely, +5200 km/sec.

Examples of the second type, in which the bright lines predominate, are obtained from emission patches in the outer regions of large nebulae, for instance, N.G.C. 5457 (M 101) and N.G.C. 6822.

Wide, shallow absorption lines have been observed in high-dispersion spectra of M 31 and M 32, and have been found approximately twice as wide in M 32 and almost four times as wide in M 31 as the lines in the spectrum of skylight. A widening of the lines seems noticeable in spectra of the fainter nebulae, but no definite

statement in regard to this can be made, on account of the small scale.

The maximum of the continuous spectrum is shifted by an amount equal to the displacement due to the velocity. This suggests a color excess, which has actually been found from extra-focal photographic and photovisual magnitudes and which in some cases exceeds the amount expected. Nebulae in the Perseus cluster have an excess color-index of half a magnitude. Indications are that color-index appears to depend upon galactic latitude rather than distance.

#### FUTURE INVESTIGATIONS

An attempt will be made to extend the observed range in distance by measures of fainter clusters of nebulae. Some extension seems quite possible, but the limit with the 100-inch reflector will be reached at about photographic magnitude 17.5. Exposures necessary for the fainter nebulae are not so long as the magnitudes would indicate because the red-shift is so large that the H and K lines are brought into the region to which the photographic plate is highly sensitive. Further, lower dispersion can be used, for, since the red-shift is larger, a larger probable error can be tolerated. The main difficulty arises from the fact that at photographic magnitude  $17.5 \pm$  the nebulae become so faint visually at the Cassegrain focus of the 100-inch reflector that they cannot be seen on the slit of the spectrograph.

In order to test thoroughly the agreement of the velocities for individual members of a cluster, a larger number of velocities will be observed in the Virgo cluster.

High-dispersion spectra of some of the brightest nebulae will be obtained in order to investigate further the widening of the absorption lines which appears in such objects as M 31 and M 32.

I wish to express my thanks to Mr. T. A. Nelson, night assistant at the 100-inch telescope, and to Mr. Glenn Moore, relief night assistant, for their aid in obtaining the spectrograms.

CARNEGIE INSTITUTION OF WASHINGTON  
MOUNT WILSON OBSERVATORY  
March 1931

## THE VELOCITY-DISTANCE RELATION AMONG EXTRA-GALACTIC NEBULAE<sup>1</sup>

By EDWIN HUBBLE AND MILTON L. HUMASON

### ABSTRACT

Methods of determining distances of extra-galactic nebulae are discussed, and the mean absolute magnitude is revised on the basis of (1) Shapley's revision of the zero-point of the period-luminosity curve for Cepheids, and (2) more extensive observations of stars involved in nebulae. The revised value is  $M(\text{vis}) = -14.9$ .

The mean color-index of the nearer extra-galactic nebulae appears to be of the order of  $+1.1$  mag., hence  $M(\text{pg}) = -13.8$ . A color-excess is suggested which is independent of distance but shows some relation to galactic latitude.

The velocity-distance relation is re-examined with the aid of 40 new velocities, 26 of which refer to nebulae in 8 clusters or groups. Distances of the clusters, ranging out to about 32 million parsecs, have been derived from the most frequent apparent magnitudes. The velocity displacements reduce the apparent magnitudes by amounts which become appreciable for the more distant clusters.

The new data extend out to about eighteen times the distance available in the first formulation of the velocity-distance relation, but the form of the relation remains unchanged except for the revision of the unit of distance. The relation is

$$\text{Vel.} = \frac{\text{Dist. (parsecs)}}{1790},$$

and the uncertainty is estimated to be of the order of 10 per cent.

### PART I. DISTANCES OF NEBULAE

Distances of nebulae are derived from the application of absolute-magnitude criteria to stars involved in the nebulae. This is the only direct method available at present, and it plays the same rôle among the nebulae that micrometer measures play among the stars. All other methods are calibrated by means of the small sample collection of nebulae in which stars can be seen and studied. The assumption that this collection is a representative sample is a first approximation, supported in a general way by the consistency of the results to which it leads. The basic data will be materially increased only when larger telescopes and faster plates are available.

Absolute-magnitude criteria can be applied only when stars of familiar types can be identified. Such stars are by no means the brightest in the nebulae, hence the method is restricted to the very

<sup>1</sup> *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 427.

nearest of the neighboring systems. There is evidence, however, for a fairly definite upper limit to the absolute luminosity which stars attain, and this permits estimation of the distances of all nebulae in which any stars can be seen, even when particular types cannot be identified. Probable errors are considerable, but statistical results are reliable and are valuable in enlarging the sample collection of nebulae with distances derived from familiar criteria.

When no stars can be seen, a new criterion is required, necessarily calibrated by the data for the sample collection. This is furnished by the absolute luminosities of the nebulae themselves, which exhibit a restricted range about a well-defined most frequent value. The criterion is statistical, but when it is once calibrated, its application appears to be quite general.

#### DISTANCES DERIVED FROM STARS OF RECOGNIZED TYPES

Stellar types which have been identified in nebulae include novae, Cepheid variables, irregular variables, helium stars (Bo and O, sometimes involved in emission luminosity), and P Cygni stars. The Cepheids furnish the most reliable distances; the other types are important as confirming the general order of distance. Distances are expressed in terms of a unit defined by the zero-point of the period-luminosity relation among Cepheids; hence revisions of the zero-point change the value of the unit without affecting the relative distances of the nebulae. Shapley's<sup>1</sup> recent revision reduces the previously current value of the unit by about 11 per cent. The new value may be accepted as of the right general order, although further revision may be expected from the data on the motions of Cepheids which are now being accumulated.<sup>2</sup>

Including the Magellanic Clouds, there are 8 nebulae in which types of stars have been identified, and these, together with the 2 companions of M 31, give 10 nebulae with distances derived from the criteria of absolute magnitude. These are the most reliable distances available at present, but 3 of them, those for M 81, M 101,

<sup>1</sup> *Star Clusters*, p. 189, 1930. Revised distances are given for the Large Magellanic Cloud, 26,200 parsecs,  $m-M=17.10$ , and for the Small Magellanic Cloud, 29,000 parsecs,  $m-M=17.32$ .

<sup>2</sup> Gerasimovič (*Astronomical Journal*, 41, 17, 1931) suggests a correction of the order of 1 mag. on the basis of motions at present available.

and N.G.C. 2403, are less certain than the others.<sup>1</sup> The distances, total absolute visual magnitudes, and, for later use, absolute photographic magnitudes of the brightest stars involved in these 10 nebulae are listed in Table I. The distances and stellar magnitudes, the

TABLE I  
DISTANCES OF EXTRA-GALACTIC NEBULAE

System	Distance in Parsecs*	$M_n$	$M_s$
LMC.....	$0.290 \times 10^5$	-16.6	-5.8
SMC.....	0.262	15.8	7.4
N.G.C. 6822.....	1.92	12.0	5.6
M 33.....	2.36	14.9	6.3
M 31.....	2.47	17.0	5.8
M 32.....	2.47	13.2	.....
N.G.C. 205.....	2.47	12.7	.....
M 101.....	4.0	13.1	6.0
N.G.C. 2403.....	(6.3)	15.3	6.0
M 81.....	(7.3)	-16.0	-5.8
Mean $M_s$ .....	.....	.....	-6.1
$m_s - m_n$ from Table II.....	.....	.....	+8.9
Mean $M_n$ .....	.....	-14.7	-15.0
Adopted $M$ (vis).....	.....	.....	-14.9
CI.....	.....	.....	1.1
$M$ (pg).....	.....	.....	-13.8

\* Distances are corrected for Shapley's revision (*Star Clusters*, p. 189, 1930) of the zero-point of the period-luminosity curve for Cepheids.

latter on the international scale, were determined at Mount Wilson, except for the Magellanic Clouds, for which Shapley's results are used. The total visual magnitudes are based upon Holetschek's measures as corrected by Hopmann,<sup>2</sup> except those for the Clouds,

<sup>1</sup> A study of the 20 plates of M 101 available at Mount Wilson indicates 3 probable novae with a mean observed maximum of  $18.0 \pm$  as compared with 17.1 in M 31; 2 short-period variables, probably Cepheids, with maxima about 19.0 as compared with 18.0 for the brightest Cepheids in M 31 and M 33; 3 apparently irregular variables with maxima between 18.0 and 18.5 as compared with maxima of irregular variables in M 31 and M 33 ranging from 15.3 to 18.9. These results suggest a modulus for M 101 about 1 mag. greater than for M 31 and M 33, corresponding to a distance about 60 per cent greater. The distances of M 81 and N.G.C. 2403 are suggested by a comparison of 7 irregular variables in the former and 5 in the latter with those in M 31 and M 33.

<sup>2</sup> *Astronomische Nachrichten*, 214, 425, 1921.

estimated from various sources, and that for N.G.C. 6822, measured at Mount Wilson.<sup>1</sup>

The range in total absolute magnitude is about 5.0, with an average residual of 1.5 around the mean value of  $-14.7$ . The range in the magnitudes of the brightest stars is about 1.8, with an average residual of 0.4 around the mean value  $-6.1$ . The scattering among the brightest stars is considerably smaller than that among the nebulae themselves and would be almost negligible were it not for the outstanding case of the Large Magellanic Cloud.

UPPER LIMIT OF STELLAR LUMINOSITY  
AS A CRITERION OF DISTANCE

These facts lend color to the assumption of a fairly uniform upper limit to the luminosity of stars in the great isolated systems, which may be used as a criterion of distance where stars can be seen but no types can be identified. This assumption is reasonable in the light of current ideas on stellar constitution, and empirical investigations, beyond the range of Table I, indicate that it is fairly reliable, at least in a statistical sense. If the brightest stars have a uniform intrinsic luminosity, the scattering in the differences in apparent magnitude between the nebulae and their brightest stars should represent the scattering in the absolute magnitudes of the nebulae alone. In other words, let the subscripts  $n$  and  $s$  refer to the nebula as a whole and to the brightest stars, respectively; then if  $M_s$  is constant, the scattering in  $m_s - m_n$  should represent that in  $M_n$ . Conversely, if  $M_s$  varies through a considerable range, the scatter in  $m_s - m_n$  should be conspicuously greater than the scatter in  $M_n$  alone. Actually, the available data support the first of these alternatives. The scattering in the observed  $m_s - m_n$  is fully accounted for by that in  $M_n$ , as indicated both by Table I and also by the several clusters of nebulae where, regardless of the distance, the scattering in  $M_n$  is represented by the scattering in  $m_n$  in each cluster.

<sup>1</sup> An additional system, I.C. 1613, is an irregular nebula similar to the Magellanic Clouds and N.G.C. 6822. The nature of this system was discovered by W. Baade at Hamburg, who communicated the information by letter. The nebula is very faint and highly resolved. Several variables, some with Cepheid characteristics, have already been found, but further investigation is necessary in order to settle the matter. Preliminary estimates indicate an upper limit for the stars of the order of  $-6$  and hence are in this respect thoroughly consistent with the previous results.

The relevant data are listed in Table II and exhibited graphically in Figure 1. With one known exception,<sup>1</sup> resolution into stars is restricted to spirals of intermediate and late types, whether normal or barred, and to the irregular nebulae. Only these types were considered. Moreover, in order that the data should be fairly representative and homogeneous, the observations were restricted to nebulae of visual magnitude 11.5 or brighter on Holetschek's scale as corrected by Hopmann. There are about 140 such objects in the entire sky, of which 76 have been measured by Holetschek. Photographs

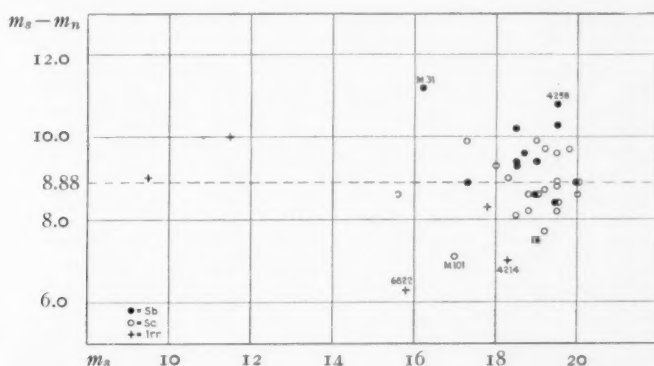


FIG. 1.—Independence of  $m_s - m_n$  and  $m_s$ , where  $m_n$  is the apparent magnitude of a nebula and  $m_s$  that of the brightest stars in the nebula. Since the scatter in  $m_s - m_n$  is fully accounted for by the known range in the absolute magnitudes of the nebulae, the range in the absolute magnitudes of the brightest stars is negligible, and  $m_s$  may be used as a criterion of relative distance.

with the large reflectors at Mount Wilson are available for 67 of these, which, together with N.G.C. 6822 and the Magellanic Clouds, represent about 50 per cent of the entire list. Except for the three objects just mentioned, the visual magnitudes of the nebulae,  $m_n$ , in Table II are those of Holetschek,<sup>2</sup> while the photographic magnitudes of the brightest stars,  $m_s$ , represent measures or estimates from the plates for the brightest four or five objects which appeared to be individual stars. Stars were found in 40 nebulae, or about 57 per

<sup>1</sup> M 87, an apparently globular nebula in the Virgo cluster. Stars cluster around the periphery, the brightest being about 19.5. A nova has been observed among them.

<sup>2</sup> Actually they are Holetschek's measures with Hopmann's scale corrections applied (see *Mt. Wilson Contr.*, No. 324; *Astrophysical Journal*, 64, 321, 1926).

TABLE II  
APPARENT MAGNITUDES OF NEBULAE AND THEIR  
BRIGHTEST STARS

N.G.C.	$m_s$	$m_n$	$m_s - m_n$
Type Sb			
224.....	16.2	5.0	11.2
1068.....	18.7	9.1	9.6
2841.....	> 19.5	9.4	> 10.1
3031.....	18.5	8.3	10.2
3310.....	> 19.0	10.4	> 8.6
3489.....	> 20.0	11.2	> 8.8
3623.....	> 20.0	9.9	> 10.1
3627.....	18.5	9.1	9.4
3628.....	> 20.0	11.4	> 8.6
4030.....	19.5	11.1	8.4
4192.....	> 19.5	10.9	> 8.6
4210.....	> 19.5	10.8	> 8.7
4258.....	19.5	8.7	10.8
4438.....	> 19.0	10.3	> 8.7
4450.....	> 20.0	10.6	> 9.4
4565.....	> 19.5	11.0	> 8.5
4736.....	17.3	8.4	8.9
4826.....	18.5	9.2	9.3
5055.....	19.0	9.6	9.4
5746.....	> 19.5	10.4	> 9.1
7331.....	19.0	10.4	8.6
Mean (10).....			9.58
Type SBb			
3351.....	> 19.5	11.4	> 8.1
3414.....	> 19.5	11.5	> 8.0
3504.....	> 20.0	11.4	> 8.6
4245.....	> 20.0	11.1	> 8.9
4314.....	> 20.0	11.1	> 8.9
4394.....	> 19.0	11.5	> 7.5
4548.....	20.0	11.1	8.9
4699.....	> 19.5	10.0	> 9.5
4725.....	19.5	9.2	10.3
5566.....	> 19.0	11.1	> 7.9
Mean (2).....			9.6
Type Sc			
253.....	18.3	9.3	9.0
598.....	15.6	7.0	8.6
628.....	18.8	10.6	8.2
1084.....	20.0	11.4	8.6
2403.....	18.0	8.7	9.3
2683.....	> 20.0	9.9	> 10.1
2903.....	19.0	9.1	9.9
3147.....	> 19.0	11.4	> 7.6
3521.....	19.8	10.1	9.7
3810.....	19.5	11.3	8.2

TABLE II—Continued

N.G.C.	$m_s$	$m_n$	$m_s - m_n$
Type Sc—Continued			
4088.....	19.0	11.5	7.5
4254.....	19.0	10.4	8.6
4321.....	19.2	10.5	8.7
4414.....	> 19.5	10.1	> 9.4
4490.....	18.8	10.2	8.6
4501.....	> 19.0	10.5	> 8.5
4559.....	19.5	10.7	8.8
4569.....	> 19.5	10.9	> 8.6
4605.....	19.5	9.9	9.6
4631.....	19.2	9.5	9.7
5005.....	19.5	11.1	8.4
5104.....	17.3	7.4	9.9
5236.....	18.5	10.4	8.1
5248.....	19.2	11.5	7.7
5457.....	17.0	9.9	7.1
6503.....	> 19.0	9.9	> 9.1
Mean (20).....			8.71

## Type SBc

613.....	> 19.5	10.6	> 8.9
3953.....	20.0	11.1	8.9
4303.....	19.5	10.6	8.9
4579.....	> 19.5	9.7	> 9.8
Mean (2).....			8.90

## Type Irr

LMC.....	9.5	0.5	9.0
SMC.....	11.5	1.5	10.0
3034.....	> 20.0	9.0	> 11.0
4214.....	18.3	11.3	7.0
4449.....	17.8	9.5	8.3
4656.....	19.0	11.5	7.5
4753.....	> 20.0	11.4	> 8.6
5363.....	> 20.0	11.1	> 8.9
6822.....	15.8	9.5	6.3
Mean (6).....			8.02

## MEANS

Type	No.	$m_s - m_n$
Sb.....	12	9.58
Sc.....	22	8.73
Irr.....	6	8.02
Total.....	40	8.88

cent of the number observed. When no stars could be detected, the estimated limit of the photograph is given with a suitable sign indicating that stars, if present, are fainter than the given  $m_s$ . In general, these limits are above the extreme limits of the plates on account of the difficulties of detecting stars on a luminous background.

The range in  $m_s - m_n$  is 4.9 mag., and the average residual is 0.77 around the mean value 8.88. The range corresponds with that of  $M_n$  in Table I and in the various clusters, about 5.0, or slightly more. The average residual is considerably less than that for the nebulae in Table I, but is closely comparable with the mean residuals found in the clusters, about 0.8 mag. The distribution of  $m_s - m_n$  is fairly symmetrical about the mean, and forms a curve similar to the distribution of apparent magnitudes in clusters of nebulae. Table I indicates no conspicuous relation between  $M_n$  and  $M_s$ , and Figure 1 no relation<sup>1</sup> between  $m_s - m_n$  and  $m_s$ . The extreme values of  $m_s - m_n$ , 11.2 for M 31 and 6.3 for N.G.C. 6822, refer to systems whose distances are well known and are obviously explained by the extreme values of  $M_n$ . These considerations indicate that the range in  $M_s$ , if not negligible, is at least much less than the range in  $M_n$ , and hence justify the use of  $M_s$  as a criterion of distance. The limiting magnitudes, where no stars were detected, are thoroughly consistent with this interpretation.

On account of the restricted data in Table I, the evaluation of  $M_s$  is less certain. The simple mean, -6.1, together with the mean  $m_s - m_n$ , 8.9, in Table II gives a mean  $M_n = -15.0$  as compared with -14.7 for the 10 nebulae in Table I. The brighter value may be given double weight because of the small range in  $M_s$  and the relatively extensive data for  $m_s - m_n$ . For this reason the mean absolute visual magnitude of extra-galactic nebulae is adopted as

$$M_n (\text{vis}) = -14.9.$$

<sup>1</sup> The differences  $m_s - m_n$  show a conspicuous correlation with the nebular magnitudes,  $m_n$ , fainter than about 9.5, in the sense that the differences decrease as the nebular magnitudes increase. This is an effect of selection due to the considerable range in the absolute magnitudes of the nebulae. Stars can be detected with reasonable certainty down to about 19.5, corresponding to a normal nebula of magnitude about 10.6. Among nebulae which appear fainter, stars can be seen only in those which are intrinsically faint and hence nearer than the average, in which case the differences  $m_s - m_n$  are necessarily small.

Hence,

$$M_s \text{ (pg)} = -6.0.$$

The corresponding formula for the distance of a nebula, as indicated by the brightest stars involved, is

$$\log D = 0.2m_s + 2.2,$$

where the distance,  $D$ , is expressed in parsecs. The uncertainty in  $D$  is probably of the order of 20 per cent.

One further point should be mentioned in connection with Table II. The nebulae are listed according to type, and the mean differences  $m_s - m_n$  appear to vary systematically with the type, the means being 9.6, 8.7, and 8.0 for *Sb*, *Sc*, and *Irr.*, respectively. The data are very limited, and the effect can be attributed largely to 3 *Sb* nebulae, M 31, N.G.C. 4258, and N.G.C. 4725, and to 3 *Irr.* nebulae, N.G.C. 4214, 4656, and 6822. The former are known or suspected to be unusually bright, and the latter unusually faint. Their elimination would remove most of the systematic effect, but the procedure would be rather arbitrary. Further investigation, including nebulae in the Virgo cluster, supports the systematic variation, but reduces the amount very considerably. The subject will be discussed in a later paper. The evidence indicates both a slight decrease in the average luminosity of nebulae and a slight increase in the luminosity of the brightest stars along the sequence of classification. These, however, are second-order effects and can be ignored in preliminary statistical investigations.

The use of the brightest stars as a criterion of distance has been criticized on the ground that clusters and groups at such remote distances would not be distinguished from single stars. The criticism appears reasonable, but in actual practice there seems to be little or no confusion. The data for the nearer nebulae behave in the same manner as those for the more distant, and the frequency distribution of the differences  $m_s - m_n$  is fairly consistent with available information concerning the luminosity function of the nebulae themselves. It is possible that this homogeneity is accidental, but its value for empirical investigations is nevertheless considerable.

TOTAL LUMINOSITIES OF NEBULAE AS A  
CRITERION OF DISTANCE

For nebulae in which no stars can be seen, the only criteria of distance are the apparent dimensions and magnitudes of the nebulae themselves. The dimensions have only a limited application. They vary systematically throughout the sequence of classification; hence it is necessary that the nebulae be photographed on a scale sufficient to show the detailed structure on which the classification is based. Moreover, since the luminosity in general fades away from the nuclei, dimensions increase systematically with effective exposure and at rates varying, among other things, with type and nuclear luminosity. This applies also to the total luminosities of nebulae—increasing exposure brings new regions above the threshold of the plates—but since the luminosities are largely concentrated in the nuclear regions, the percentage effect is much less serious than in the case of dimensions. It frequently happens that moderate exposures on faint nebulae register diameters only one-third those which full exposures reveal, while two-thirds of the total luminosities are registered. The resulting effects on distances as derived from apparent diameters and from luminosities are about 300 and 20 per cent, respectively. Such considerations indicate that, while the diameter may be used to advantage in special cases,<sup>1</sup> the total apparent magnitude offers much the more reliable criterion of distance. Luminosity, moreover, varies only slightly along the sequence of classification, and, as previously remarked, this second-order effect can be ignored in general statistical investigations. The greatest uncertainty at present arises from the lack of a well-established system of nebular photometry, free from serious systematic errors, over the observed range in apparent magnitude.

## VISUAL MAGNITUDES

The range in the absolute luminosities of nebulae, from Tables I and II and from the clusters, appears to be of the general order of 5 mag. with an average residual of about 0.8 mag. around a mean value provisionally adopted as  $-14.9$  on Holetschek's visual scale.

<sup>1</sup> Provisional mean values for the dimensions of nebulae along the sequence of classification are given in *Mt. Wilson Contr.*, No. 324; *Astrophysical Journal*, **64**, 321, 1926. The revised value of  $M_n$  reduces the tabulated values by about 11.5 per cent.

In view of his long experience in the visual photometry of comets, Holetschek's measures form the most complete and homogeneous list of nebular magnitudes available at the present time. The scale as revised by Hopmann<sup>1</sup> appears to be reasonably free from serious systematic errors as far as they have been checked, that is, to about 12.0. To the same limit the list of nebulae is reasonably complete or representative. Accidental errors naturally occur, but for statistical investigations these are of slight importance.

#### PHOTOGRAPHIC MAGNITUDES

Photographic photometry of focal nebular images is unsatisfactory for two reasons. Luminosity determined from such images varies with the effective exposure at a rate depending on several factors, and the photometry of surfaces differs widely from that of point-source images. Approximate estimates may be derived from simple comparisons of focal images when these are comparable in size with the images of stars, but the procedure involves a rather obscure subject, namely, the manner in which the photometry of surfaces merges into that of point-source images. As far as the writers know, this field has never been investigated in detail. The complications are avoided by using extra-focal images larger than the focal dimensions of the nebulae. These images are closely comparable with images of the comparison stars, and the resulting luminosities are then independent of exposure. The method has been used for the Mount Wilson magnitudes, both photographic and photovisual, referred to in the following pages. The magnitudes are on the international scale and were derived with the 10-inch Cooke astrographic camera (f/4.5) and with the large reflectors. With the 60-inch reflector, the Ross zero-power correcting lens, which eliminates coma over a field about  $1^\circ$  in diameter, proved of very great value, especially for the brighter nebulae where suitable comparison stars are rather widely scattered.

Among the very few published magnitudes of nebulae derived from photographic plates are the two lists from Harvard. One<sup>2</sup> con-

<sup>1</sup> Hopmann measured the magnitudes of the comparison stars for which Holetschek had used the *B.D.* magnitudes.

<sup>2</sup> *Proceedings of the National Academy of Sciences*, 15, 565, 1929.

sists of 47 nebulae for which radial velocities were then available; the second<sup>1</sup> consists of nearly 2800 nebulae in the region of the Virgo cluster, repeating magnitudes of 103 of the brighter objects previously published<sup>2</sup> in *Harvard Circular* No. 294 with a scale revision for a few fainter than 13.0. Both lists purport to give photographic magnitudes on the international scale as derived from direct comparisons between focal images of nebulae and of stars. Except for the fainter nebulae in the Virgo cluster, small-scale plates were used. Five of the Virgo nebulae are common to the two lists, but a systematic difference averaging about 1 mag. indicates some unexplained factor, and the lists must be examined separately.

Of the shorter list, 15 objects have been measured at Mount Wilson on extra-focal plates with a thermocouple photometer. Comparison stars were standardized by reference to the North Polar Sequence and to Selected Areas. The results are given in Table III. The differences  $MW-H$  range through some 3 mag. and average about  $+0.5$ , which probably accounts for the mean color-index of  $+0.23$  derived by comparing Harvard photographic with Holetschek's visual magnitudes. The results indicate the present unsatisfactory state of photographic nebular photometry and the necessity for standard conventions. The writers believe that the latter must be based on extra-focal methods.

The list of magnitudes in the Virgo cluster appears to be much more homogeneous, at least for the brighter nebulae. Extra-focal magnitudes, both photographic and photovisual, have been measured

<sup>1</sup> *Harvard Annals*, 88, 1930.

<sup>2</sup> Wirtz in *Publicationen der Sternwarte in Kiel*, No. 15, p. 35, 1927, gives magnitudes of 98 nebulae in the Virgo cluster measured with an electrophotometer from small-scale, focal plates. He derived his zero-point and scale from the Harvard nebular magnitudes in *Circular* No. 294 and hence his results merely confirm the general similarity of the focal images as observed with the eye and with the photometer. The method of calibration accounts for the absence of systematic differences in Wirtz's results from short and long exposures. Baade (*Mitteilungen der Hamburger Sternwarte in Bergedorf*, 6, 98 [No. 29], 1928) gives photographic magnitudes of 25 nebulae in the Ursa Major cluster, ranging from 16.0 to 18.0, measured on plates with the 1-m. f/3 reflector. The method is not stated, but, since the nebulae are very small and faint, the conditions were suitable for focal comparisons. The Mount Wilson extra-focal measures agree with Baade's results from 16.0 to 17.3, beyond which the former are somewhat brighter. There is no marked improvement over Baade's method, whatever that may have been. No other lists of photographic magnitudes are known to the writers.

for about 60 nebulae in the central region of the cluster. The differences  $MW-H$  vary systematically, but not linearly, through a range of about 0.6 mag. from 10.5 to 14.0, beyond which the extra-focal measures are not yet sufficiently numerous for comparison. Twenty of the brighter nebulae are in Holetschek's list and furnish the following means:

$$MW(pg) - Harv(pg) = +0.15 \text{ (ave. res., 0.2) ,}$$

$$MW(pv) - Hol(vis) = -0.07 \text{ (ave. res., 0.35) .}$$

TABLE III  
MAGNITUDES OF BRIGHT NEBULAE

N.G.C.	$MW_{pg}$	$S_{pg}$	$H_{vis}$	$MW-S$	$MW-H$
205.....	11.0	11.2	9.3	-0.2	+1.7
221.....	9.6	8.7	8.8	+0.9	+0.8
278.....	11.8	11.7	12.0	0.1	-0.2
404.....	12.0	11.6	11.1	0.4	+0.9
584.....	11.8	11.8	10.9	0.0	0.9
936.....	11.6	11.2	11.1	0.4	0.5
1023.....	11.4	11.2	10.2	0.2	1.2
1068.....	9.85	9.8	9.1	0.0	0.7
1700.....	12.7	12.1	12.5	0.6	0.2
4382.....	10.9	9.7	10.0	1.2	0.9
4472.....	10.1	9.1	8.8	1.0	1.3
4486.....	10.8	9.2	9.7	1.6	1.1
4526.....	11.3	10.3	11.1	1.0	0.2
4649.....	10.8	9.8	9.5	+1.0	1.3
7619.....	13.1	14.6	11.8	-1.5	+1.3
Means.....				+0.45	+0.85

$MW$  = Mount Wilson extra-focal measures.

$S$  = Shapley, *Proceedings of the National Academy of Sciences*, 15, 569, 1929.

$H$  = Holetschek.

#### COLOR-INDICES

The mean color-index from extra-focal measures of the 60 nebulae in the Virgo cluster is  $+1.10 \pm 0.02$ . This must be compared with  $+0.9 \pm 0.12$  from a comparison of  $MW(pg)$  with Holetschek's visual magnitudes for 10 nebulae in Table III, and with values ranging from  $+1.05$  to  $+1.2$  for the brighter members of four clusters of fainter nebulae derived from extra-focal measures at Mount Wilson, and finally with the value of  $+1.3$  for Wolf's cluster in Perseus

(gal. lat.  $-13^\circ$ ). These measures indicate a mean color-index of the order of  $+1.1$  for nebulae in general, and hence a mean absolute photographic magnitude of the order of  $-13.8$ .

The spectral types exhibit a narrow range about the mean G<sub>3</sub>, and large-scale spectra of M<sub>31</sub> and M<sub>32</sub> indicate definite dwarf characteristics. Thus the color-indices suggest a color-excess<sup>1</sup> of the order of 0.3 mag. Since the effect exhibits no conspicuous dependence on distance and the largest value is in the lowest galactic latitude, the source is possibly within the galactic system itself. This conclusion must be accepted with appropriate reservations, however, for although there is no evidence whatever for absorption, either selective or general, in extra-galactic space, there still remains the possibility of undetected systematic effects in the measures or even unrecognized characteristics in the nebulae themselves. The apparent velocity displacements, for the nearer nebulae at least, can account for only a small fraction of the observed color-excess.<sup>2</sup> Further investigations are under way which should either correct the measures or throw light on the contents of interstellar space. Meanwhile, provisional values of the mean absolute magnitudes of nebulae,  $-14.9$  pv and  $-13.8$  pg, may be used for distance criteria provided the apparent magnitudes are based upon extra-focal exposures.

The luminosity criterion is purely statistical and is reliable only when large numbers of nebulae are available. One direct application concerns the great clusters of nebulae. The mean or most frequent apparent magnitude of the many members is a good indication of the distance of a cluster, and hence clusters offer the greatest distances that can definitely be assigned to individual objects. If observations of very remote objects are desired, the brightest members of very faint clusters may be selected. This procedure assumes that cluster nebulae are comparable with isolated nebulae, but evidence from several sources indicates that the assumption is well founded.

<sup>1</sup> The color-excess is in general agreement with Humason's observation that the distribution of intensities in the continuous spectra corresponds to spectral types considerably later than those indicated by the absorption lines.

<sup>2</sup> This effect is discussed in a later section of the present paper.

## PART II. VELOCITY-DISTANCE RELATION

## FIRST FORMULATION

The criteria of distance, approximately in the form described, have been known for several years. With their aid a relation between radial velocity and distance was established among the nebulae for which velocities were known. The relation is a linear increase in the velocity amounting to about  $+500$  km/sec. per million parsecs of distance. This result was published<sup>1</sup> two years ago, together with the announcement that a program was under way having for its purpose the testing of the relation over as great a range in distance as could be covered with existing equipment. The first result of the new program, a velocity for N.G.C. 7619 in the Pegasus cluster which was consistent with the relation already established, was mentioned in the paper.<sup>2</sup>

## NEW DATA

About forty new velocities are now available, representing in general very faint and hence distant nebulae, and the velocity-distance relation may be examined in the light of the new material. The spectrograms were photographed by Humason and measured by Humason and Miss MacCormack, with the exception of nine which were obtained by Pease and very kindly placed at our disposal for measurement and discussion. These spectra have been discussed separately,<sup>3</sup> hence only the displacements, expressed as velocities, will be considered here.

Velocities previously available, owing very largely to the great

<sup>1</sup> Hubble, *Mt. Wilson Communication*, No. 105; *Proceedings of the National Academy of Sciences*, **15**, 168, 1929. The revision of  $M_n$  increases the coefficient to about 550 km/sec. per million parsecs.

<sup>2</sup> Professor de Sitter (*Bulletin of the Astronomical Institutes of the Netherlands*, **5**, 157 [No. 185], 1930) published a redetermination of the velocity-distance relation in which he arrived at the same numerical result. This was to be expected since he used essentially the same data, together with a few new velocities published from Mount Wilson, some accompanied by estimates of the distances in order to emphasize the consistency of the results from the new program. There are certain criticisms of the treatment of the data (for instance, the use of the brightest members of clusters as nebulae of normal luminosity, etc.), but these are of minor importance. Differences tend to cancel out, and the final numerical result agrees with the Mount Wilson result.

<sup>3</sup> Humason, *Mt. Wilson Contr.*, No. 426; *Astrophysical Journal*, **74**, 35, 1931.

pioneer work of V. M. Slipher at the Lowell Observatory, although naturally restricted to the brighter, nearer nebulae, included five members of the Virgo cluster. This set the limit to the observed range in distance used for the first formulation of the velocity-distance relation. The new program contemplated the use of much greater distances, far beyond the limits at which stars can be detected, and hence included the several brightest members of as many clusters or groups as possible. Isolated nebulae were also included to fill gaps in the observing seasons and to furnish a few mean points that might test the similarity of isolated and cluster nebulae.

The new data are listed in Table IV. Observed displacements, expressed as velocities, are reprinted from Humason's paper.<sup>1</sup> Holetschek's visual magnitudes are available for 6 isolated nebulae; photographic magnitudes from extra-focal measures are given for 10 isolated nebulae; while for 8 clusters or groups the most frequent magnitude is listed. The magnitude for the Virgo cluster is assumed to be 12.5. Corrections for the solar motion were computed from data given in *Mt. Wilson Communication* No. 105,<sup>2</sup> and the corrected apparent velocities of the nebulae were rounded off to the nearest 50 km/sec. There is no advantage in recomputing the solar motion with the new material, for the uncertainties in the distances, although relatively moderate, are numerically so large as to mask the effect which is sought.

#### ISOLATED NEBULAE

The isolated nebulae have been combined into two groups: the 6 nebulae having Holetschek magnitudes and the 10 having photographic magnitudes (including N.G.C. 4865 for reasons which will be discussed later). These give the mean values  $\log v = 3.109$  for  $m = 11.75$  vis, or  $+1280$  km/sec. for 2.15 million parsecs; and  $\log v = 3.534$  for  $m = 14.3$  pg, or  $+3420$  km/sec. for 4.2 million parsecs, respectively. The first is in fair agreement with the results for the

<sup>1</sup> Three nebulae in Humason's list are omitted: N.G.C. 205, a companion to M 31; N.G.C. 6822; and M 101. The last two were included in the first presentation of the velocity-distance relation, the first being there omitted only by an oversight. Its inclusion with M 31 and M 32 does not affect the results.

<sup>2</sup> *Proceedings of the National Academy of Sciences*, **15**, 168, 1929.

TABLE IV  
VELOCITIES AND MAGNITUDES

	$v$	$v_s$	$v_o$	$m_{vis}^*$	$m_{pg}$	Type	Diam.
Virgo cluster:	km/sec.						
N.G.C. 4192...	+ 1150	+ 20					
4374...	1050	30					
4382...	500	40					
4472...	850	20	+ 890	.....	(12.5)	.....	.....
4486...	800	30					
4526...	580	20					
4649...	1090	+ 40					
Pegasus:							
N.G.C. 7611...	3400						
7617...	3900						
7619...	3800	+ 90	3810	.....	15.5	.....	.....
7623...	3800						
7626...	3700						
Pisces:							
N.G.C. 380...	4400						
383...	4500	+ 60	4630	.....	15.4	.....	.....
384...	4500						
385...	4900						
Cancer:							
N.G.C. 2562...	5100	- 130	4820	.....	16.0	.....	.....
2563...	4800						
Perseus:							
N.G.C. 1270...	4800						
1273...	5800	0	5230	.....	16.4	.....	.....
1275...	5100						
1277...	5200						
Coma:							
N.G.C. 4853...	7600						
4860...	7900	+ 100	7500	.....	17.0	.....	.....
4865†...	5000						
4884...	6700						
Ursa Majoris:							
Baade 24.....	11700	+ 100	11800	.....	18.0	.....	.....
Leo:							
No. 1.....	19700	- 90	19600	.....	19.0	.....	.....
Isolated nebulae:							
N.G.C. 2859...	1500	- 50	1450	11.1	.....	SBa	1.9
2950...	1500	+ 60	1560	11.6	.....	SBa	1.4
3193...	1300	- 60	1240	12.1	.....	E2	1.0
3227...	1150	- 60	1090	12.0	.....	Sb	3.0
3610...	1850	+ 100	1950	11.8	.....	E4	1.4
4051...	650	+ 90	740	11.9	.....	Sb	4.0
6359...	3000	+ 250	3250	.....	14.3	E2	0.4
6658...	4100	+ 270	4370	.....	14.8	Sa	1.6
6661...	3900	+ 270	4170	.....	14.0	Sa	1.6
6702...	2250	+ 280	2530	.....	14.6	E2	0.6
6703...	2000	+ 280	2280	.....	13.6	E0	0.9
6710...	5100	+ 280	5380	.....	15.0	Sa	0.8
6824...	3200	+ 240	3440	.....	14.0	Sb	1.6
7217...	1050	+ 200	1250	.....	12.3	Sb	3.0
7242...	+ 5000	+ 200	+ 5200	.....	15.5	E2	0.3

\* Holetschek visual magnitudes.

† Possibly an isolated nebula superposed on the cluster.  $m_{pg}=14.7 \pm$ . The type is E4; the diameter, 0'.13.

clusters, but the second is discrepant by about 1000 km/sec. or 1 mag. The logarithms of the velocities have been used in deriving the mean values because magnitudes are logarithmic functions of distances.

#### CLUSTERS OF NEBULAE

*Virgo cluster* (R.A.  $12^h 25^m \pm$ , Dec.  $+12^\circ 5' \pm$ , 1930; gal. long.  $256^\circ$ , lat.  $+73^\circ$ ).<sup>1</sup>—The cluster includes several hundred members scattered over an elliptical area about  $12^\circ \times 10^\circ$ . It is the largest and nearest of the known clusters and includes 16 out of the 34 extragalactic objects in Messier's list. Nebulae of all types except the irregular are represented among its members, but elliptical nebulae and early spirals are relatively much more numerous than among the nebulae at large. The predominance of early types is a conspicuous feature of clusters in general, and the Virgo cluster is exceptional in the considerable number of late-type spirals which are included.

This cluster has been investigated at the Harvard College Observatory, where the 24-inch Bruce camera is especially suited to the problem. The system of magnitudes there established appears to deviate systematically from the Mount Wilson extra-focal magnitudes, but between 12.0 and 12.5 the two systems are nearly the same. For this reason 12.5, adopted at Harvard as the most frequent photographic magnitude, is used in the present discussion. A study of the cluster has been under way with the large reflectors at Mount Wilson, but the angular extent is so great that the data are not yet complete. The distance provisionally adopted is 1.8 million parsecs. This result, which differs widely from Shapley's estimate of 10 million light-years,<sup>2</sup> is derived from the following sources:

a) The most frequent apparent magnitude, 12.5, combined with the adopted mean absolute magnitude,  $-13.8$ , gives 1.8 million parsecs.

b) Stars with magnitudes as indicated have been found in the following nebulae:

<sup>1</sup> H. Shapley and Adelaide Ames have recently reported an extension of the Virgo cluster, stretching many degrees to the south (*Harvard Bulletin*, No. 880, 1930).

<sup>2</sup> Or about 9 million light-years on the revised scale.

N.G.C. 4254.....Sc 18.8	N.G.C. 4303.....Sbc 19.5
4321.....Sc 19.0	4567.....Sc 20.0
4535.....Sc 19.3	4568.....Sc 20.0
4294.....Sc 19.5	4548.....SBb 20.0
4298.....Sc 19.5	4486*.....Eo 19.5
4713.....Sc 19.5	

\*N.G.C. 4486 is the only known example of an elliptical nebula in which stars can be detected.

Since other late-type nebulae in the cluster show no stars as bright as 20.0, the mean value for the upper limit is probably not brighter than 20.0 and, in view of the restricted range in the upper limit suggested by Tables I and II, is probably not much fainter than 20.0. The adopted  $M_s = -6.0$ , combined with apparent limits of 20.0 and 20.5 for stars in the cluster, gives distances of 1.6 million and 2.0 million parsecs, respectively. It is probable that the actual distance lies between these limits. From these considerations it is concluded that the stars involved indicate a distance of the order of 1.8 million parsecs.

c) The Virgo cluster offers a special case in which apparent diameters can be used as a criterion of distance with some degree of reliability. The analysis of the cluster is still incomplete, but the mean diameters may be approximated from the known values for the largest nebulae, together with probable values for the smallest as estimated by inspection of the many plates available. This procedure leads to Table V, in which the diameters in parsecs are from a previous investigation,<sup>1</sup> but have been reduced by 11.5 per cent to agree with the revised value for  $M_n$ .

The uncertainties in the method are obvious and the precise value of the result has little weight; yet the order of the distance is probably reliable and agrees with the previous indications. When the complete analysis of the cluster with the large reflectors is available, the mean apparent diameters will be revised in the light of the frequency distributions of the diameters and a closer approximation will be made.

The seven velocities now available for the Virgo cluster exhibit a total range of 550 km/sec. about the mean value +890 km/sec.

*The Pegasus cluster* (R.A.  $23^h 17^m$ , Dec.  $+7^\circ 50'$ , 1930; gal. long.

<sup>1</sup> *Mt. Wilson Contr.*, No. 324; *Astrophysical Journal*, 64, 321, 1926.

$55^\circ$ , lat.  $-49^\circ$ ).—This cluster consists of about 100 nebulae scattered over an area roughly  $1^\circ$  in diameter about the two brightest objects, N.G.C. 7619 and 7626. Of these, the former is the brighter, with a total photographic magnitude of 13.1. From this upper limit the magnitudes range down to about 18.0, with the most frequent value at 15.5. Since the faint end of the frequency-curve is rather uncertain, the peak, i.e., the most frequent luminosity, is used here, as in all the other clusters, as the criterion of relative distance. Extra-focal measures of the brighter nebulae obtained with the 10-inch

TABLE V  
DIAMETERS AND DISTANCES

TYPE	DIAMETERS			DISTANCE
	Largest	Smallest	Mean	
E0-E2.....	2'.0	0'.1	1'.05 = 380 parsecs	$1.3 \times 10^6$ parsecs
E3-E5.....	3.0	0.1	1.55 = 620	1.4
E6-E7.....	2.4	0.2	1.3 = 1000	2.7
Mean E.....				$1.8 \times 10^6$
Sa.....	5.0	0.2	2.6 = 1280	1.7
Sb.....	8.0	0.3	4.15 = 1680	1.4
Sc.....	6.1	0.4	3.25 = 2210	2.4
Mean S.....				$1.8 \times 10^6$
Mean E and S.....				$1.8 \times 10^6$

camera and the 60-inch reflector are in good agreement, and color-indices derived from photovisual magnitudes are available for N.G.C. 7619 and 7626. These are 1.1 and 1.0, respectively, but are subject to some uncertainty. The distance, as indicated by the most frequent apparent magnitude, is 7.25 million parsecs. Velocities for five nebulae show a total range of 500 km/sec. around the mean value  $+3800$  km/sec.

*The Pisces group* (R.A.  $1^h35^m$ , Dec.  $+32^\circ$ , 1930; gal. long.  $96^\circ$ , lat.  $-30^\circ$ ).—This is not a cluster, but a group of some 25 elliptical nebulae which stands out from the approximately uniform background of field nebulae. The brighter ones are catalogued as N.G.C. 379, 380, 382, 383, 384, and 385. The luminosities, as indicated by the hump on the otherwise smooth curve expressing the frequency

distribution over a field about  $1^\circ$  in diameter, range through some 3 mags. and are rather unsymmetrical about the most frequent magnitude, 15.4. This corresponds to a distance of 7 million parsecs. The small number of objects introduces a corresponding uncertainty into this result and possibly accounts for the restricted range in luminosity. The four velocities available range through 500 km/sec., with a mean around +4630 km/sec.

*The Cancer cluster*<sup>1</sup> (R.A.  $8^h16^m5$ , Dec.  $+21^\circ20'$ , 1930; gal. long.  $170^\circ$ , lat.  $+30^\circ$ ).—This consists of about 150 nebulae distributed over an area of nearly 1 square degree centered near N.G.C. 2562. The cluster has not been analyzed in so detailed a manner as the others, but the range is normal and the most frequent magnitude is about 16.0, a value which is definitely established to the nearest half-magnitude. The corresponding distance is 9 million parsecs. No color-indices have been measured. Velocities of 2 nebulae are available, averaging about +4800 km/sec.

*The Perseus cluster* (R.A.  $3^h15^m$ , Dec.  $+41^\circ15'$ , 1930; gal. long.  $118^\circ$ , lat.  $-13^\circ$ ).—A cluster of about 500 nebulae scattered over an area nearly  $2^\circ$  in diameter, discovered by M. Wolf<sup>2</sup> in 1905. The center is near N.G.C. 1275, the brightest nebula in the cluster, with a photographic magnitude about 13.8. The nebulae are elliptical for the most part, but include an occasional early-type spiral, either normal or barred. Magnitudes range from 13.8 to about 19.0, with a most frequent magnitude rather sharply defined at 16.4, thus indicating a distance of 11 million parsecs.

Internally consistent color-indices for 16 nebulae have been derived from extra-focal measures, both photographic and photovisual, on two pairs of plates made with the 60-inch reflector. Comparison stars were standardized by means of Selected Areas and the North Polar Sequence. For 15 nebulae the color-indices range from 1.2 to 1.4, with a mean of about 1.3. The photographic magnitudes extend from 14.6 to 16.7 and appear to be independent of the colors. The

<sup>1</sup> The Cancer cluster was discovered independently by Dr. Edwin F. Carpenter, of the Steward Observatory, Tucson, Arizona, who communicated the information by letter and later presented a paper on the cluster at the Pasadena meeting of the A.A.A.S. in June, 1931. The cluster was first photographed at Mount Wilson in January, 1929.

<sup>2</sup> *Astronomische Nachrichten*, 170, 211, 1905; *Veröffentlichungen der Sternwarte zu Heidelberg*, 6, 131, 1913.

remaining nebula, N.G.C. 1275, has a color-index of about 0.75, a discrepancy which was probably explained when a strong emission spectrum was registered from the bright stellar nucleus.<sup>1</sup> The unusually large color-indices of the other nebulae appear to be confirmed by an exposure-ratio plate made by the method developed by Seares. The measures suggest excessive colors for all the 60 nebulae recorded, but precise quantitative interpretation requires a careful calibration which has not yet been effected. An abnormally high color-excess appears, however, to be definitely established. In view of the low galactic latitude,  $-13^\circ$ , and the position bordering on a known region of calcium absorption, the color-excess suggests scattering by material within the galactic system. In this case the photographic magnitudes should appear fainter than normal and the distance derived from them should be too great. A comparison of this distance, 11 million parsecs, with that derived from the velocity-distance relation and the mean velocity of +5200 km/sec. from 4 nebulae indicates a discrepancy in the proper direction and of about the proper amount.<sup>2</sup> The deviation is small, however, and not definitely outside the limits of certainty or consistency in the correlation of velocities and distances. The results, in short, are suggestive but not definitive.

*The Coma cluster* (R.A.  $12^h 55^m 5$ , Dec.  $+28^\circ 20'$ , 1930; gal. long.  $26^\circ$ , lat.  $+87^\circ$ ).—The cluster consists of about 800 nebulae scattered over an area roughly  $1.7$  in diameter, centered near N.G.C. 4874 and 4884. Some 25 of the brightest nebulae were observed visually by D'Arrest<sup>3</sup> as early as 1865, but the nature and extent of the cluster were not fully recognized until Wolf<sup>4</sup> photographed the region in

<sup>1</sup> N.G.C. 1275 is classed as an irregular nebula but not of the Magellanic Cloud type. It appears to be somewhere between the elliptical nebulae and the spirals. For purely descriptive purposes it could be classed as an elliptical nebula which has broken up without the formation of spiral arms.

<sup>2</sup> The most frequent photographic magnitude, 16.4, indicates a distance of 11 million parsecs and hence a velocity of about +6000 km/sec. Since the color-index is 0.2 mag. greater than that for nebulae in general, a correction of 0.4 mag. is suggested by the hypothesis of scattering. This reduces the most frequent magnitude to 16.0, the distance to 9.1 million parsecs, and the expected velocity to +5000 km/sec. The observed velocity is +5200 km/sec.

<sup>3</sup> *Astronomische Nachrichten*, 65, 1, 1865.

<sup>4</sup> *Ibid.*, 155, 127, 1901.

1901 and recorded 108 nebulae in a circle 30' in diameter. H. D. Curtis,<sup>1</sup> with the Crossley reflector, counted 304 in an area 40'  $\times$  50', and on a plate by J. C. Duncan with the 100-inch reflector, over 400 have been found in about the same area. Photographic magnitudes range from about 14.1 to 19.5, with 17.0 as the most frequent value. The apparent velocity displacement, as will be shown later, indicates a correction of  $-0.1$  to the apparent photographic magnitude. The corresponding distance is then about 13.8 million parsecs. Color-indices from two pairs of extra-focal exposures average 1.12 for 10 of the brighter nebulae.

The analysis of this cluster will be described in some detail as an example of the method used for them all. The adjacent regions were first photographed in order to delineate the cluster and to determine the number of nebulae included. Next, by focal comparisons with Selected Areas and the North Polar Sequence, a standard sequence of comparison stars was established in the cluster. Then, from extra-focal exposures, on which the images were of various dimensions, a sequence of nebular magnitudes was established. Finally, on all the photographs available, from small camera plates to those with the 100-inch reflector, the nebulae were counted in a specified area; in this particular cluster, about 1500 square minutes. For the counts, a long exposure with the 100-inch was used as a guide in order that no nebula might be missed or mistaken for a star on the shorter exposures and the small-scale plates. Each plate thus gave the number of nebulae to a definite limit, which was determined from the sequence of extra-focal magnitudes extending to about 18.1. Fainter magnitudes were estimated by extrapolations guided by previous results from statistical investigations of the limits corresponding to various exposure conditions. The numbers of nebulae to successive limits were corrected for field nebulae by tables derived from unpublished investigations of the relation between number of nebulae and effective exposure or limiting magnitude over the higher galactic latitudes.

The data for the Coma cluster are listed in Table VI and exhibited in Figure 2 as a relation between  $N_m$ , the number of nebulae to a given limiting magnitude, and  $m$ , the limiting magnitude itself.

<sup>1</sup> *Publications of the Lick Observatory*, 13, 33, 1918.

From the smooth curve in Figure 2 the numbers of nebulae within successive intervals of magnitude were derived from the differences in the ordinates. These gave the frequency distribution represented in Figure 3, in which the lower curve represents the numbers in successive intervals of 0.5 mag.; the upper curve, on a reduced scale, gives the sums for three such intervals and thus represents a smoothing by overlapping means. In spite of the uncertainty in the fainter

TABLE VI  
DISTRIBUTION OF MAGNITUDES IN THE COMA CLUSTER

Limit $m_{pg}$	Nebulae	Source	Interval	Nebulae*
14.5 . . . . .	1	Measured magnitudes	14.0-14.5	1
15.0 . . . . .	4	Measured magnitudes	14.5-15.0	3
15.5 . . . . .	17	Measured magnitudes	15.0-15.5	14
16.2 . . . . .	60	Measured magnitudes	15.5-16.0	29
16.5 . . . . .	80	10-inch focal	16.0-16.5	34
17.2 . . . . .	140	100-inch extra-focal	16.5-17.0	43
17.3 . . . . .	150	60-inch extra-focal	17.0-17.5	38
18.1 . . . . .	202	100-inch extra-focal	17.5-18.0	34
19.2 . . . . .	255	36-inch focal†	18.0-18.5	29
19.5 . . . . .	266	100-inch focal	18.5-19.0	23
20.5 . . . . .	263	100-inch focal	19.0-19.5	15
			19.5-20.0	2
			20.0-20.5	0

\* Numbers were read from a smooth curve representing the data in the first two columns.

† Photograph with the Crossley reflector, available through the courtesy of the director of the Lick Observatory.

magnitudes, the curves are very symmetrical. The peak is at 17.0,<sup>1</sup> with an uncertainty of probably not more than 0.1 mag.

Velocities for 4 nebulae are available. Three of these, N.G.C. 4853, 4860, and 4884, show a range of 1200 km/sec. about the mean velocity +7500 km/sec. The fourth, N.G.C. 4865, has a velocity of +5100 km/sec. It is an elliptical nebula, about E4, estimated at 14.7 mag., similar in appearance to the other cluster nebulae. From an inspection of the plates it would be described simply as one of the brightest members of the cluster. Its velocity, however, is the one outstanding discrepancy among some 28 velocities in 8 clusters or groups. This suggests that it is a field nebula, seen in projection

<sup>1</sup> This value is 0.1 mag. brighter than that given for the Coma cluster in the *Annual Report of the Mount Wilson Observatory, 1929-1930*, and represents a revision of the system of magnitudes.

against the cluster. The absolute photographic magnitude, as derived from the velocity-distance relation, would then be  $-15.1$ , nearer the mean luminosity of nebulae in general than the value  $-16.0$  which must be assigned if it is considered a member of the cluster. Such cases must occasionally be encountered and this may be an in-

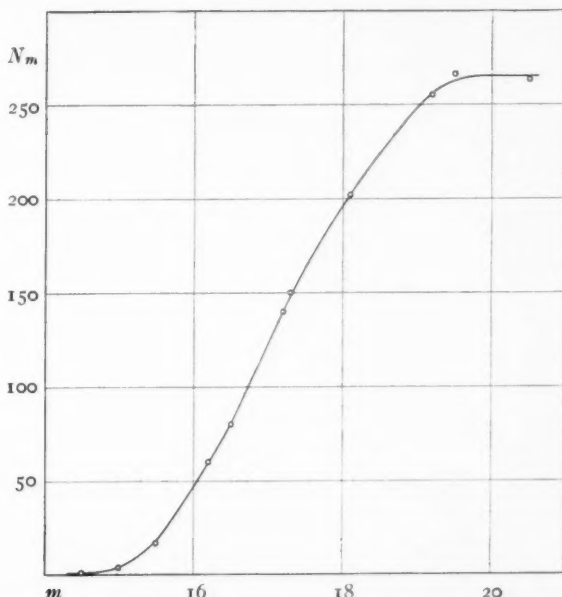


FIG. 2.—Counts of nebulae in the central region of the Coma cluster.  $N_m$  is the number of nebulae as bright or brighter than apparent photographic magnitude  $m$ .

stance. Observations of the cluster will be continued, however, especially in view of the wide range in the three more normal velocities, in order that the matter may be re-examined in the light of more data.<sup>1</sup>

<sup>1</sup> Humason has since obtained velocities for 5 additional nebulae in the Coma cluster: 6600, 6900, 6900, 7000, and 8500 km/sec. for I.C. 4045 and N.G.C. 4872, 4874, 4881, and 4895, respectively. The uncertainties are rather large, since each velocity is derived from a single spectrogram with the small dispersion of 875 Å per millimeter. For this reason the range, 1900 km/sec., may be exaggerated, but it is clearly larger than those observed in the other clusters and raises the question of a possible correlation between range and distance.

The mean velocity of the 8 nebulae is 7360, as compared with 7500 for the 3 nebulae previously observed. The effect of this revision on the correlation of velocities and distances is considerably less than the probable errors.

*The Ursa Major cluster* (R.A.  $11^h43.3^m$ , Dec.  $+56^\circ8'$ , 1930; gal. long.  $106^\circ$ , lat.  $+59^\circ$ ).—This cluster comprises about 300 nebulae within a roughly circular area about  $0.7$  in diameter. The cluster was discovered by W. Baade,<sup>1</sup> who published a description, together with photographic magnitudes of the brighter nebulae, in 1928. It was found independently at Mount Wilson in 1924 and delineated by photographs covering the adjacent regions. The magnitudes range from about 16.0 to fainter than 20.0. The most fre-

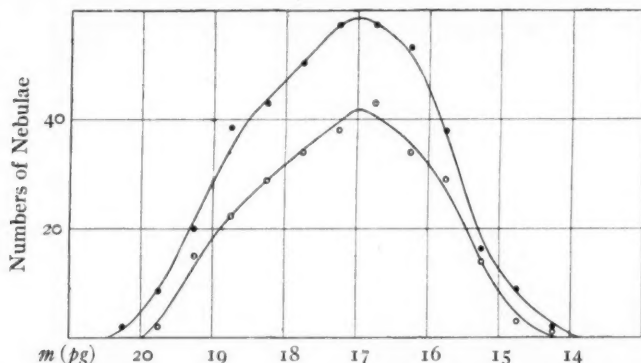


FIG. 3.—Frequency distribution of apparent photographic magnitudes among nebulae in the central region of the Coma cluster. The lower curve (open circles) represents the numbers of nebulae in successive intervals of 0.5 mag. as read from the curve in Fig. 2. The upper curve, on another scale, represents a smoothing by overlapping means.

quent magnitude is about 18.0, which, corrected by  $-0.15$  mag. to compensate for an effect of the red-shift, corresponds to a distance of 22 million parsecs. Extra-focal magnitudes from two pairs of plates give a mean color-index of  $+1.20$  for 10 nebulae; an exposure-ratio plate gives a mean of  $+1.16$  for 20 nebulae. The extra-focal photographic magnitudes agree very closely with Baade's values for nebulae brighter than about 17.3, beyond which the extra-focal measures are somewhat brighter. By comparing the brighter nebulae in the two clusters Baade derived a distance of the order of fifteen times that of the Virgo cluster. This is only 25 per cent greater than the relative distances adopted in the present discussion, although Baade's absolute distance, based upon Shapley's value of 10 million

<sup>1</sup> *Loc. cit.*

light-years for the Virgo cluster, is twice that derived from the most frequent magnitude.

A velocity has been measured for nebula No. 24 in Baade's list. Before the spectrogram was obtained, a velocity of +12,000 km/sec. was predicted on the basis of the distance indicated by the most frequent magnitude. The measures, corrected for solar motion, give +11,800 km/sec.

*The Leo cluster* (R.A.  $10^h 24^m$ , Dec.  $+10^\circ 50'$ , 1930; gal. long.  $201^\circ$ , lat.  $+54^\circ$ ).—This cluster, which includes about 300 nebulae scattered over an area approximately  $0.6$  in diameter, was called to our attention by W. H. Christie, who discovered it on plates taken with the 60-inch reflector in December, 1929. The faintest nebulae are probably beyond the limits of the photographs available, and the form of the frequency-curve of apparent magnitudes is difficult to determine. Both the brightest and the most frequent magnitude, determined independently, are about 1 mag. fainter than those in the Ursa Major cluster, and this agreement lends added confidence to the value assigned to the Leo cluster, namely, 19.0 for the most frequent magnitude. The correction necessitated by the red-shift in the spectrum is about 0.25 mag.; hence the distance is of the order of 32 million parsecs.

One small-scale spectrogram (about 875 Å to the millimeter) has been obtained for one of the brightest nebulae in the cluster. The definition is excellent, and the five lines identified, including the all-important H and K lines, establish a displacement corresponding to +19,600 km/sec. beyond reasonable doubt. No color measures are available as yet.

#### EFFECT OF RED-SHIFTS ON APPARENT MAGNITUDES

If nebular luminosity approximates black-body radiation, the red-shifts produce redistributions of intensities which can be represented by new black-body curves corresponding to lower temperatures and hence to later spectral types. This affects the observed photographic magnitudes by amounts which become appreciable for  $d\lambda/\lambda > 0.02$ , corresponding to distances greater than, say, 10 million parsecs.

The relation between total radiation and photographic magnitude is obtained as follows: The total radiation of a celestial body is ex-

pressed by the bolometric magnitude  $m_b$ . Allowance for absorption by the earth's atmosphere,  $\Delta m_r$ , gives the radiometric magnitude,  $m_r$ , the quantity measured by a thermocouple:

$$m_r = m_b + \Delta m_r .$$

Visual magnitude is related to radiometric magnitude and heat-index,  $HI$ , just as photographic magnitude is related to visual magnitude and color-index,  $CI$ . Thus

$$m_{vis} = m_r + HI ,$$

$$m_{pg} = m_{vis} + CI .$$

Therefore

$$m_{pg} = CI + HI + \Delta m_r + m_b .$$

Bolometric magnitudes are derived from the observed radiometric magnitudes by calculating the correction  $\Delta m_r$  on the assumption of black-body radiation. Color- and heat-index are observed quantities which bear known empirical relations to spectral types as derived from absorption lines where the displacement of the lines is negligible. Under these conditions both indices and spectral types are related to temperature. Deviations of the indices from the normal values corresponding to spectral types thus determined are commonly referred to as color-excess,  $CE$ , and heat-excess,  $HE$ , respectively. They represent abnormal distribution of intensity in the continuous spectrum as compared with the pattern of absorption lines.

A red-shift, by redistributing the radiation to correspond with a lower temperature and hence with a later spectral type, introduces an increment to the photographic magnitude which, by virtue of the last equation, can be expressed as

$$\begin{aligned} \Delta m_{pg} &= \Delta CI + \Delta HI + \Delta[\Delta m_r] + \Delta m_b , \\ &= CE + HE + \Delta[\Delta m_r] + \Delta m_b . \end{aligned}$$

The first three terms on the right are evaluated from the advance in spectral type, i.e., they represent the differences between the in-

dices and  $\Delta m_r$  for the type as indicated by the absorption lines, dG3 in the case of the nebulae, and the type corresponding to the lower apparent temperature introduced by the red-shift.

From Wien's law the new temperature is

$$T = \frac{5760}{1 + \frac{d\lambda}{\lambda}},$$

where 5760 is the temperature for dG3 and  $d\lambda/\lambda$  is the red-shift. The corresponding spectral type and hence the color-excess, heat-excess, and  $\Delta[\Delta m_r]$  are then derived graphically from the data in Table VII.

TABLE VII

Spectral Type	Temp.*	Color-Index†	Heat-Index‡	Correction to No Atmosphere‡
F5.....	6500°	0.62	0.30	0.44
dGo.....	6000	.72	.32	.43
dG5.....	5600	.83	.39	.41
dKo.....	5100	0.99	0.55	.40
dK5.....	4400	1.26	1.10	.48
dM.....	3400	1.76(Ma)	1.40(Mo)	0.53(Mo)

\* Russell, Dugan, and Stewart, *Astronomy*, 2, 753, 1927.

† Seares, *Mt. Wilson Contr.*, No. 226; *Astrophysical Journal*, 55, 165, 1922. Color-indices are in Table XII.

‡ Pettit and Nicholson, *Mt. Wilson Contr.*, No. 360; *Astrophysical Journal*, 68, 270, 1928. Heat-indices are from Table V; corrections to no atmosphere are from Table IV (in which the temperatures are those in Table V corresponding to water-cell absorptions).

The increment in the bolometric magnitude follows from the decrease in  $\nu$  introduced by the red-shift. The energy of each quantum of radiation is reduced by a constant factor, namely,  $1 + d\lambda/\lambda$ , and hence the total radiation, i.e., the sum of all the quanta, is reduced by the same factor. The increment in  $m_b$  is therefore

$$\Delta m_b = 2.5 \log \left( 1 + \frac{d\lambda}{\lambda} \right).$$

If an actual velocity of recession is involved, an additional increment, equal to that given above, must be included in order to account for the difference in the rates at which the quanta leave the

source and reach the observer.<sup>1</sup> This additional increment will be neglected for the present, since the writers desire to emphasize the observational results without discussing the interpretation more than is necessary for the immediate purpose. The increments, moreover, are small; for the largest observed red-shift,  $\Delta m_b$  is about 0.07 mag., and the effect on the final correlation is less than the probable error. Eventually it may be possible to test the matter by counts of nebulae to successive limits of apparent magnitude on the assumption of uniform distribution, an assumption which appears to be well established to about the limits at which the additional increment would be expected to become sensible.

Table VIII gives the temperature, spectral type, color-excess, heat-excess reduced to no atmosphere, and the increments in the bolometric and the photographic magnitudes corresponding to different red-shifts in a normal dG3 spectrum. Approximate distances as indicated by the velocity-distance relation are added in the last column.<sup>2</sup> These results ignore the color-excess, independent of apparent velocity displacements, which the photometric observations appear to indicate. If further investigations confirm the preliminary indications, the quantities in Table VIII must be revised.

The effect of the red-shift on the photographic magnitude is about 0.25 for the Leo cluster, 0.15 for the Ursa Major cluster, and 0.10 for the Coma cluster. These corrections were applied in calculating the distances from the observed magnitudes. For objects nearer than the Coma cluster, the corrections are less than the uncertainties of the magnitudes and have been ignored.

<sup>1</sup> The factor is  $\sqrt{\frac{1+v/c}{1-v/c}}$ , which closely approximates  $1+d\lambda/\lambda$  for red-shifts as large as have been observed. A third effect due to curvature, negligible for distances observable at present, is discussed by R. C. Tolman in *Proceedings of the National Academy of Sciences*, 16, 511, 1930.

<sup>2</sup> Table VIII extends the calculations of  $\Delta m_{pg}$  out to red-shifts about three times those actually observed in order to indicate the possibility of testing the velocity-distance relation near the limits of direct photography with large telescopes. If the relation is approximately linear, the limit of the 100-inch reflector for normal nebulae should correspond to a red-shift of the order of 40,000 km/sec. The proposed 200-inch may be expected to approach 60,000 km/sec. The values of  $\Delta m_{pg}$  are 0.55 and 0.95, respectively, and should be readily detected in the correlation of numbers of nebulae per plate and exposure times, provided the assumption of uniform distribution is approximately true.

## RELATION BETWEEN RED-SHIFTS AND APPARENT MAGNITUDES

The quantities actually observed in the present investigation are red-shifts and apparent magnitudes. The relation between them is so definite and significant that it may be emphasized before interpreting the magnitudes in terms of distance. The fact that the red-shifts are expressed on a scale of velocities is incidental; for the present purpose they might as well be expressed as  $d\lambda/\lambda$ .

TABLE VIII

EFFECT OF VELOCITY DISPLACEMENTS ON PHOTOGRAPHIC MAGNITUDES

Velocity Km/Sec.	$d\lambda/\lambda$	Temp. Abs. Cent.	Spectral Type	Color- Excess	Heat- Excess*	$\Delta m_b$	$\Delta m_{pg}$	Distance in Parsecs
1000.....	0.0033	5740	dG 3.3	0.008	0.004	0.003	0.015	$1.8 \times 10^6$
4000.....	.0133	5685	4.0	.02	.015	.015	.05	7.2
8000.....	.0267	5615	4.7	.04	.03	.03	.10	14.4
12000.....	.0400	5540	5.8	.06	.05	.04	.15	21.6
16000.....	.0533	5470	6.5	.08	.06	.06	.20	28.8
20000.....	.0667	5400	7.2	.10	.08	.07	.25	36
30000.....	.100	5235	8.8	.16	.13	.11	.40	54
40000.....	.133	5080	K 0.2	.21	.20	.14	.55	72
50000.....	.167	4940	1.4	.26	.32	.17	.75	90
60000.....	0.200	4800	2.3	0.31	0.44	0.20	0.95	108

\* Reduced to no atmosphere.

The significant data are listed in Table IX. For comparison with the clusters, the 16 isolated nebulae are combined in a single group, giving a mean  $\log v$  corresponding to a mean magnitude, the 6 visual magnitudes being reduced to the photographic system by the adopted color-index, +1.1. Another mean for isolated nebulae is furnished by the 21 objects in which no stars can be seen, which are listed in the former discussion of the velocity-distance relation.<sup>1</sup> The visual magnitudes are reduced to the photographic system as above. This group was not used directly in the preliminary formulation, which was based primarily upon nebulae in which stars could be identified. Its inclusion in the present discussion does not materially affect the results, but it permits the statement that the present correlation in-

<sup>1</sup> *Mt. Wilson Communication*, No. 105; *Proceedings of the National Academy of Sciences*, 15, 168, 1929. The 21 unresolved nebulae are those in Table II, with the exception of N.G.C. 4526, which belongs to the Virgo cluster and was included in the table by an oversight.

cludes all nebulae with observed velocities for which apparent magnitudes are the sole criterion of distance.

TABLE IX

Cluster	Number of Nebulae	Diameter	Mean Velocity	Number of Velocities	Mean $m_{pg}$	$\Delta m^*$	Color-Index
Virgo.....	(500)	12°	890	7	12.5	.....	1.1
Pegasus.....	100	1	3810	5	15.5	.....	1.1
Pisces.....	20	0.5	4630	4	15.4	.....	.....
Cancer.....	150	1.5	4820	2	16.0	.....	.....
Perseus.....	500	2.0	5230	4	16.4	.....	1.3
Coma.....	800	1.7	7500	3	17.0	-0.10	1.1
Ursa Major....	300	0.7	11800	1	18.0	0.15	1.2
Leo.....	400	0.6	19600	1	19.0	-0.25	.....
Isolated nebulae I	.....	.....	2350	16†	13.8	.....	.....
Isolated nebulae II	.....	.....	630	21‡	11.6	.....	.....

\* Correction due to apparent velocity displacement (see Table VII).

† Isolated nebulae in Table IV. Velocity represents the mean  $\log v$ . Visual magnitudes are corrected for color-index.

‡ Unresolved isolated nebulae previously observed (Table II in *Proceedings of the National Academy of Sciences*, 15, 171, 1929). Velocity represents the mean  $\log v$ ; visual magnitudes are reduced to photographic.

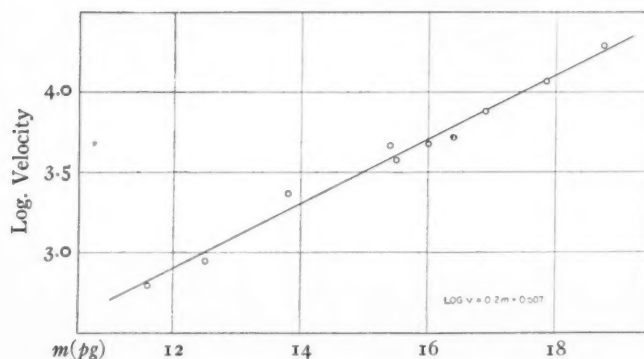


FIG. 4.—Correlation between the quantities actually observed in deriving the velocity-distance relation. Each point represents the mean of the logarithms of the observed red-shifts (expressed on a scale of velocities) for a cluster or group of nebulae, as a function of the mean or most frequent apparent photographic magnitude.

In Figure 4 the mean logarithms of the velocities are plotted against the most frequent or mean apparent magnitudes for the clusters and the two groups of isolated nebulae. Each point is given

unit weight. The correlation is closely linear and a least-squares solution leads to the equation

$$\log v = (0.202 \pm 0.007)m + 0.472. \quad (1)$$

This justifies the adoption of the exact decimal 0.2 as the coefficient of  $m$ ; a new solution then gives

$$\log v = 0.2m + 0.507 \pm 0.012. \quad (2)$$

The average deviation from this formula is 0.031 in  $\log v$  and 0.15 in  $m$ , over a range of 1.5 in  $\log v$  and 7.25 in  $m$ .

Since distance in parsecs is expressed by

$$\log d = \frac{m - M + 5}{5} = 0.2m - 0.2M + 1,$$

equation (2) may be written

$$\log \frac{v}{d} = 0.2M - 0.493. \quad (3)$$

Also, since

$$M = c - 2.5 \log L,$$

where  $L$  is the intrinsic luminosity,

$$\log \frac{v}{d} = c_1 - 0.5 \log L,$$

$$\frac{v}{d} = \frac{c_2}{\sqrt{L}}.$$

Hence, independent of theory, it follows that the ratio of red-shift to distance is constant, provided the intrinsic luminosity of nebulae is statistically constant; otherwise the ratio varies inversely as the square root of the intrinsic luminosity.

#### VELOCITY-DISTANCE RELATION

The constancy of  $M_n$  is believed to be well established, hence apparent magnitudes of nebulae are accepted as measures of distance. The interpretation of red-shifts as actual velocities, however, does

not command the same confidence, and the term "velocity" will be used for the present in the sense of "apparent" velocity, without prejudice as to its ultimate significance. In this sense the relation between velocity and distance is clearly linear, and the slope is determined by the numerical value of  $M_n$ . Introducing the adopted value  $M_n = -13.8$  pg into equation (3), we have

$$\log \frac{v}{d} = -3.253,$$

$$v = \frac{d}{1790} = 558 \text{ km/sec. per million parsecs.}$$

A probable error computed in the usual way would merely indicate the consistency of the data and not the real uncertainty, which arises mainly from the possibility of systematic errors in the magnitudes, color-indices, and the adopted  $M_n$ . Accidental errors and the effect of peculiar velocities, both of individual nebulae and of clusters, are probably small in comparison. It is believed, however, that the uncertainty in the final result is definitely less than 20 per cent and probably not more than 10 per cent.

This result, which may be rounded off to 560 km/sec. per million parsecs, differs from the 500 km/sec. found in the previous discussion only by the amount corresponding to the change in  $M_n$  produced by Shapley's revision of the standard unit of distance. Without the revision the new result would differ from the old by less than 1 per cent. Since the two investigations were based upon different criteria of distance, the close agreement emphasizes the internal consistency of our present ideas concerning luminosities of nebulae.

The data used in the two investigations are combined in Figure 5, where velocities are plotted against distances, the latter all reduced to the present scale. The observations now cover a range about eighteen times that available for the preliminary investigation and approach the limit of present instrumental equipment; but the form of the correlation is essentially unchanged, except for the revision in the unit of distance, and hence the velocity-distance relation appears to be a general characteristic of the observable region of space. Aside from its cosmologic significance, the relation offers a new method of

determining distances of individual objects in which the percentage errors actually diminish with distance. This opens new possibilities for the investigation of nebulae, some of which may be mentioned at the present time.

#### THE LUMINOSITY FUNCTION OF NEBULAE

In addition to the evidence on luminosity derived from nebulae in which stars can be identified, absolute magnitudes may be found

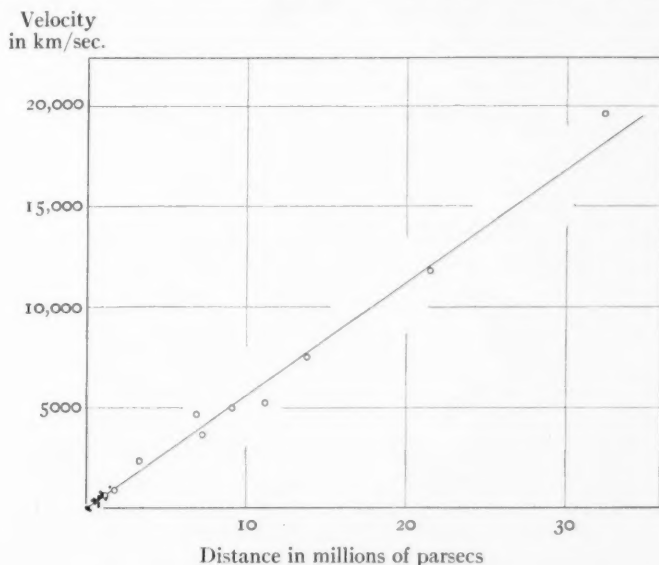


FIG. 5.—The velocity-distance relation. The circles represent mean values for clusters or groups of nebulae. The dots near the origin represent individual nebulae, which, together with the groups indicated by the lowest two circles, were used in the first formulation of the velocity-distance relation.

from the distances indicated by the apparent velocities. The number of known velocities will increase as time goes on and eventually will be sufficient to determine a reliable frequency-curve for isolated objects and even for the different types of nebulae. The study of these curves, together with those for the clusters, may be expected to throw light on several problems, including some of evolutionary significance. At present it is only possible to compare the frequency-curve for 56 isolated nebulae of all types with a combined curve for the several clusters. The result is shown in Figure 6. Distances of

the 20 isolated nebulae in which stars can be seen are derived from the stars; distances of the remaining 36 objects, from the velocities corrected for the solar motion. One object, N.G.C. 404, has been omitted because the observed velocity is so small that the peculiar motion may be large in comparison with the distance effect. The curves for the clusters are combined on the assumption that the most frequent absolute photographic magnitude is  $-13.8$ .

The curves for the clusters and for the isolated nebulae have the same range, but the former is considerably more symmetrical. It is

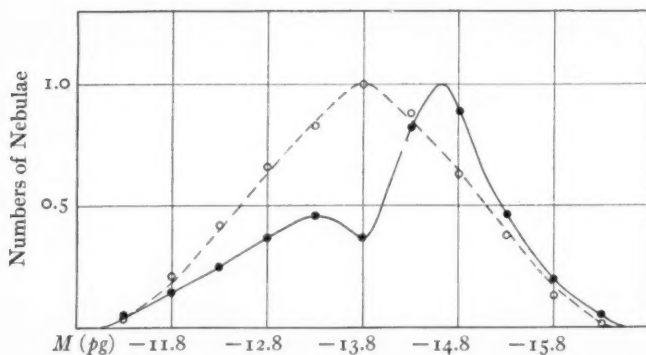


FIG. 6.—Frequency distribution of absolute photographic magnitudes among extragalactic nebulae as derived from clusters (circles) and from isolated nebulae (dots). Distances of the isolated nebulae were derived mainly from the red-shifts. The range in the two curves is the same. The asymmetry in the curve for isolated nebulae is believed to be due, in part at least, to effects of selection.

believed that the asymmetry of the curve for the isolated nebulae is partly accounted for by an effect of selection, but more extensive data will be required to determine the matter.

#### ABSORPTION OF LIGHT IN LOW GALACTIC LATITUDES

The problem of absorption within the galactic system can be approached statistically by comparing absolute luminosities of nebulae in low latitudes with those in the higher latitudes. The Perseus cluster with its large color-excess and, apparently, its low mean luminosity is very suggestive of obscuration, but it leaves the question open as to whether the material responsible for the obscuration is uniformly diffused throughout the galactic system or isolated in

clouds. Several large spirals in low latitudes are known in which the surface brightness appears abnormally faint (I.C. 342 is a good example), and the complete absence of extra-galactic nebulae along the galactic plane is generally attributed to obscuration. On the other hand, extra-galactic nebulae are found in considerable numbers near the galactic plane in longitudes  $10^{\circ}$ – $50^{\circ}$  and obscuration is not conspicuous. Several late-type spirals near I.C. 1303, in latitude  $+8^{\circ}$ , for instance, are not particularly faint for their dimensions, and exposure-ratio plates do not indicate excessive colors. The same is true

TABLE X

	$M_{ve}$	Gal. Lat.
N.G.C. 6710.....	-15.0	$+11^{\circ}$
6658.....	14.7	$+13$
6661.....	15.4	$+13$
6824.....	15.0	$+15$
7242.....	14.4	$-16$
6702.....	13.7	$+19$
6703.....	14.5	$+19$
7217.....	-14.5	$-20$
Mean.....	-14.65	

of the earlier types N.G.C. 6658 and 6661 at  $+12^{\circ}$ . Systematic investigations will be necessary in order to determine the situation, but the velocity-distance relation furnishes the method and a program is already under way. The results so far available do not favor any considerable general obscuration, but the data are meager as yet, and no definite conclusion can be drawn. The 8 nebulae within  $20^{\circ}$  of the galactic plane given in Table IV average about 0.85 mag. brighter than normal, and the brightest 4 are in the lowest latitudes. The data are given in Table X. A systematic effect of selection is probably involved. At any rate, this group of abnormally bright objects accounts for the large deviation of the mean point from that derived from the velocity-distance relation for 10 nebulae with photographic magnitudes, and for part of the asymmetry in the luminosity function shown in Figure 6.

## MASSES OF NEBULAE

The most direct indications of the masses of nebulae are derived from spectrographic rotations. The measures give linear velocities

of rotation at various angular distances from the nuclei, and in order to estimate the masses it is necessary to convert the angular distances into linear measure. This requires a knowledge of the distances of the nebulae themselves. Spectrographic rotations are now available for M 31 and M 33, that for the latter revised on the basis of new spectra of the nucleus, and for N.G.C. 4594. The distance of the last can now be derived from the apparent velocity. Further measures of rotational velocities are very desirable and will be made most easily on the early-type nebulae, E7 and Sa, in which stars are not found. With the aid of the velocity-distance relation, however, the rotation plates will themselves furnish the distances. Since the mass, for a given set of measures, varies linearly with the distance, uncertainties in the latter quantity, arising from the unknown peculiar motions of the nebulae, will not change the order of the result.

The present contribution concerns a correlation of empirical data of observation. The writers are constrained to describe the "apparent velocity-displacements" without venturing on the interpretation and its cosmologic significance. Further observations are desirable and will be carried on, although it seems probable that the general features of the relation are already sketched in outline nearly to the limit of existing equipment. Color investigations and statistical counts of nebulae to successive limits offer possibilities of extending the range of the observations while the spectrograph explores more thoroughly the region already scouted.

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## THE MAXIMUM MASS OF IDEAL WHITE DWARFS

By S. CHANDRASEKHAR

### ABSTRACT

The theory of the *polytropic gas spheres* in conjunction with the equation of state of a *relativistically degenerate electron-gas* leads to a *unique value for the mass of a star* built on this model. This mass ( $=0.91\odot$ ) is interpreted as representing the upper limit to the mass of an ideal white dwarf.

In a paper appearing in the *Philosophical Magazine*,<sup>1</sup> the author has considered the density of white dwarfs from the point of view of the theory of the polytropic gas spheres, in conjunction with the degenerate non-relativistic form of the Fermi-Dirac statistics. The expression obtained for the density was

$$\rho = 2.162 \times 10^6 \times \left( \frac{M}{\odot} \right)^2, \quad (1)$$

where  $M/\odot$  equals the mass of the star in units of the sun. This formula was found to give a much better agreement with facts than the theory of E. C. Stoner,<sup>2</sup> based also on Fermi-Dirac statistics but on uniform distribution of density in the star which is not quite justifiable.

In this note it is proposed to inquire as to what we are able to get when we use the relativistic form of the Fermi-Dirac statistics for the degenerate case (an approximation applicable if the number of electrons per cubic centimeter is  $> 6 \times 10^{29}$ ). The pressure of such a gas is given by (which can be shown to be rigorously true)

$$P = \frac{1}{8} \left( \frac{3}{\pi} \right)^{\frac{1}{3}} \cdot hc \cdot n^{\frac{4}{3}}, \quad (2)$$

where  $h$  equals Planck's constant,  $c$  equals velocity of light; and as

$$n = \frac{\rho}{\mu H(1+f)}, \quad (3)$$

<sup>1</sup> *II*, No. 70, 592, 1931.

<sup>2</sup> *Philosophical Magazine*, 7, 63, 1929.

$\mu$  equals the molecular weight, 2.5, for a fully ionized material,  $H$  equals the mass of hydrogen atom, and  $f$  equals the ratio of number of ions to number of electrons, a factor usually negligible. Or, putting in the numerical values,

$$P = K\rho^{4/3}, \quad (4)$$

where  $K$  equals  $3.619 \times 10^{14}$ . We can now immediately apply the theory of polytropic gas spheres for the equation of state given by (4), where for the exponent  $\gamma$  we have

$$\gamma = \frac{4}{3} \text{ or } 1 + \frac{1}{n} = \frac{4}{3} \text{ or } n = 3.$$

We have therefore the relation<sup>1</sup>

$$\left(\frac{GM}{M'}\right)^2 = \frac{(4K)^3}{4\pi G},$$

or

$$\begin{aligned} M &= 1.822 \times 10^{33}, \\ &= .91 \odot (\text{nearly}). \end{aligned} \quad (5)$$

As we have derived this mass for the star under ideal conditions of extreme degeneracy, we can regard  $1.822 \times 10^{33}$  as the maximum mass of an ideal white dwarf. This can be compared with the earlier estimate of Stoner<sup>2</sup>

$$M_{\max} = 2.2 \times 10^{33}, \quad (6)$$

based again on uniform density distribution. The "agreement" between the accurate working out, based on the theory of the polytropes, and the cruder form of the theory is rather surprising in view of the fact that in the corresponding non-relativistic case the deviations were rather serious.

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November 12, 1930

<sup>1</sup>A. S. Eddington, *Internal Constitution of Stars*, p. 83, eq. (57.3.)

<sup>2</sup>*Philosophical Magazine*, 9, 944, 1930.

## REVIEWS

*Publicazioni del R. Osservatorio Astronomico di Merate (Como) N. 4: Ricerche sulla frequenza delle grandzze assolute delle stelli delle diverse classi spettrali, Parte I: "Catalogo generale di parallassi stellari."* By GINO CECCHINI. Milano: Ulrico Hoepli, 1931. 4to. Pp. 152. 30 lire.

This is the most valuable contribution in its field since the publication in 1924 of Schlessinger's *General Catalogue of Stellar Parallaxes*. The author has collected the trigonometric, dynamic, and hypothetical parallaxes of 3975 stars, and reduced them, as far as possible, to a common system. As this publication is the first of a series on the relation between absolute magnitude and spectral type, the author has properly omitted the spectroscopic parallaxes, since they are deduced from the relation which he is investigating. In attempting to reduce all these parallaxes to a common system, the author first applied, to the trigonometric determinations, the corrections in the Preface of Schlessinger's *Catalogue*, which are due to the right ascensions of the stars. Next, in every case where more than one observatory has found the parallax of a star, all determinations are compared and a systematic correction found for each observatory, which is then applied to its values. The dynamical and hypothetical parallaxes are then treated in the same way, and are compared with the trigonometrical ones. The mean probable error is determined in each case.

The catalogue gives, in addition to the 1900 positions, the Henry Draper, Boss, and Cincinnati numbers. The main portion of the catalogue gives the visual magnitude and spectral type, mostly from the *Henry Draper Catalogue*; the mean corrected parallax, with the probable error and the number of determinations; the absolute magnitude; and the proper motion.

Following the catalogue there is a large table containing notes on each star, in which are given the Burnham numbers of the double stars, with their distances and position angles, and the differences in magnitude between the components; the individual determinations of the parallax where there are more than one; the maximum and minimum magnitudes of the variable stars; and any other estimates of the spectral type when these differ from those of the *Henry Draper Catalogue*.

It will be seen from this description that this is a very comprehensive and important compilation, and that much credit is due to Professor Cecchini for his work.

The second part of this work, a statistical discussion of the relations between absolute magnitude and spectral type, is in preparation.

It seems a great pity, however, that such a publication should not be deemed worthy of a better binding than the extremely inferior one in which it was put out.

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